

Review: Properties of Exponents

Basic skills Quiz 1 is over properties of exponents. You need to know the following properties:

Negative Exponents

Negative exponents indicate a reciprocal: $a^{-b} = \frac{1}{a^b}$.

Examples:

$$(3x + 4)^{-7} = \frac{1}{(3x + 4)^7}$$

$$4u^2v^{-3} = \frac{4u^2}{v^3}$$

$$\frac{3}{5t^3 - 2t + 1} = 3(5t^3 - 2t + 1)^{-1}$$

Fractional Exponents

Raising to a power which is a reciprocal of an integer is the same as taking the corresponding root:

$$a^{1/b} = \sqrt[b]{a}$$

Note that $\sqrt[b]{a}$ represents the b -th root of a ; the number that gives a when raised to the power b . So $\sqrt[3]{8} = 2$ because $2^3 = 8$. When we have a second root, we call it a square root and just leave out the 2 in the radical:

$$\sqrt[2]{16} = \sqrt{16} = 4$$

In general, we can raise to any rational power as a combination of a power and a root:

$$a^{b/c} = (a^b)^{1/c} = \sqrt[c]{a^b} = (\sqrt[c]{a})^b$$

(Technical note: The above relation holds only if all of the above are real numbers. We

cannot say for example that $(\sqrt{-1})^2 = (-1)^{2/2} = -1$ or that

$(\sqrt{-1})^2 = \sqrt{(-1)^2} = \sqrt{1} = 1$ because $\sqrt{-1}$ is not a real number. Just make sure you do not allow anything inside a radical to be negative.)

Examples:

$$\sqrt{ax + b} = (ax + b)^{\frac{1}{2}}$$

$$(z^3 - z + 7)^{\frac{3}{2}} = \sqrt{(z^3 - z + 7)^3} = (\sqrt{z^3 - z + 7})^3$$

$$\frac{1}{\sqrt[3]{(t-3)^4}} = (t-3)^{-4/3}$$

Miscellaneous Properties

You should also know the following:

$$x^0 = 1, \text{ if } x \neq 0. \text{ (} 0^0 \text{ is undefined.)}$$

$$x^1 = x \text{ for any } x.$$

In addition, be sure you know that in general $(a+b)^n \neq a^n + b^n$. (Many students make the mistake of thinking that they are equal.) In particular, note that

$$(a+b)^2 \neq a^2 + b^2$$

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}, \text{ and}$$

$$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}$$

Practice Problems

Here are a few practice problems to help you prepare. Answers are on the next page.

1. Rewrite $\sqrt{(5y)}$ in the form $(5y)^k$.
2. Rewrite $\frac{7}{(-x-6)^{3/2}}$ in the form $A(-x-6)^k$.
3. Rewrite $\sqrt[4]{(8x+4)^3}$ in the form $(8x+4)^k$.
4. Rewrite $\frac{1}{2-4x+5x^2}$ in the form $(2-4x+5x^2)^k$.
5. Rewrite $\frac{6}{(-6w-2)^{1/3}}$ in the form $A(-6w-2)^k$.
6. Rewrite $\frac{1}{\sqrt{x}}$ in the form x^k .
7. Rewrite $\frac{1}{x}$ in the form x^k .
8. Rewrite $\frac{1}{\sqrt{w+5}}$ in the form $(w+5)^k$.
9. Rewrite $\frac{3}{\sqrt{w^5}}$ in the form Aw^k .

Answers:

1. $(5y)^{1/2}$

2. $7(-x-6)^{-3/2}$

3. $(8x+4)^{3/4}$

4. $(2-4x+5x^2)^{-1}$. (Note that this is **not** the same as $2^{-1} - (4x)^{-1} + (5x^2)^{-1}$!)

5. $6(-6w-2)^{-1/3}$

6. $x^{-1/2}$

7. x^{-1}

8. $(w+5)^{-1/2}$

9. $3w^{-5/2}$