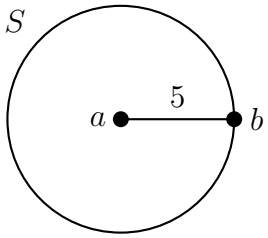
Determining How Order of Quantification Impacts Mathematical Logic

Reference:
Vroom, K. (2020). Guided reinvention as a context for investigating students' thinking about mathematical language and for supporting students in gaining fluency (Doctoral dissertation). *Dissertations and theses.*

Let S be a circle of radius 5. Let $d(a, b)$ denote the distance between two points a and b. For example, in the picture below, the point a is ***inside*** S, the point b is ***on*** S, and $d(a, b) = 5$.



1. Determine whether each of the following statements is true or false.

   (a) There is a point a inside S such that for every point b on S, $d(a, b) = 5$.
   (b) For every point b on S there is a point a inside S such that $d(a, b) = 5$.
   (c) There is a point a inside S such that for every point b on S, $d(a, b) = 7$.
   (d) For every point b on S there is a point a inside S such that $d(a, b) = 7$.
   (e) For all points a inside S, there is a point b on S such that $d(a, b) = 5$.
   (f) There is a point b on S such that for any point a inside S, $d(a, b) = 5$.
   (g) For all points a inside S, there is a point b on S such that $d(a, b) = 7$.
   (h) There is a point b on S such that for any point a inside S, $d(a, b) = 7$.

2. For which distances c, are the following statements true?

   (a) There is a point a inside S such that for every point b on S, $d(a, b) = c$.
   (b) For every point b on S, there is a point a inside S such that $d(a, b) = c$.
   (c) For all points a inside S, there is a point b on S such that $d(a, b) = c$.
   (d) There is a point b on S such that for all points a inside S, $d(a, b) = c$.