An Introduction to Multiply Quantified Statements

When an open statement has more than one variable, each variable must be quantified. Many times, each variable is quantified *differently*. Below are some examples of **multiply quantified statements**.

- There is an $x \in \mathbb{R}$ such that for every $y \in \mathbb{R}$, xy + 2x 4y 8 = y + 2
- Given any x > 0, there exists a natural number n such that $0 < \frac{1}{n} < x$.
- For every real number $\varepsilon > 0$, there is some natural number N such that for all natural numbers $n \ge N$, $|a_n L| < \varepsilon$.
- For any real number x, there is a 2×2 matrix A with det A = x.
- For some $b \in \mathbb{R}$, $a \leq b$ for all $a \in (0, 5)$.

Question: Given an open statement with more than one variable, does it matter which variable we quantify first?