Introduction to the Universal Quantifier

Consider a "for all" statement of the following form:

For all $x \in U_x$, P(x).

Notation:

Examples:

- Every real number x satisfies $x^2 > 0$.
- Given a matrix A, if det $A \neq 0$, then A is invertible.
- 0 is the additive identity on \mathbb{R} .
- \mathbb{Z} is closed under addition.

When is a "for all" statement true?

When is it false?

How might we prove it?