## Introduction to the Universal Quantifier

Consider a "for all" statement of the following form:

$$
\text { For all } x \in U_{x}, P(x)
$$

Notation:

## Examples:

- Every real number $x$ satisfies $x^{2}>0$.
- Given a matrix $A$, if $\operatorname{det} A \neq 0$, then $A$ is invertible.
- 0 is the additive identity on $\mathbb{R}$.
- $\mathbb{Z}$ is closed under addition.

When is a "for all" statement true?

When is it false?

How might we prove it?

