Introduction to the Existential Quantifier

Consider an existence statement of the following form:

There exists an $x \in U_x$ such that $P(x)$.

Notation:

Examples:

- There are real numbers x, y such that $2x + 3y = 8$.
- For some real number x, $f(x) = 0$.
- The matrix $A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$ is invertible.

When is an existence statement true?

When is it false?

How might we prove it?