Analyzing "Proofs" of a Logical Implication

Reference:

Hub, A., & Dawkins, P. C. (2018). On the construction of set-based meanings for the truth of mathematical conditionals. *The Journal of Mathematical Behavior, 50*, 90-102.

Consider the following statement.

For every integer x, if x is a multiple of 6, then x is a multiple of 3.

Below, three proofs are given. For each one, decide whether it does or does not prove the statement above. If it does not, what statement *does* it prove?

Proof 1. Consider an arbitrary integer x that is a multiple of 6. Then, $x = 6k$ for some integer k. Notice that, $x = 6k = 3(2k)$ where $2k$ is also an integer. Therefore, x is a multiple of 3. □	*Proof 2.* Consider $x = 15$. Then, $15 = 3(5)$ so x is a multiple of 3. Now suppose that $15 = 6k$ for some integer k (note: $k \neq 0$). Then, $k = 15/6$, which is not an integer. Therefore, it is impossible that $15 = 6k$ for some integer k. This means that 15 is not a multiple of 6. □	*Proof 3.* Consider an arbitrary number x that is not a multiple of 3. Suppose that this x is a multiple of 6. Then, $x = 6k$ for some integer k. This implies that $x = 3(2k)$ where $2k$ is an integer, and therefore x is a multiple of 3. Since we assumed x is not a multiple of 3, we may conclude that x cannot be a multiple of 6. □