

Proving Existence Nonconstructively

How might the given theorems be used to prove the following claim?

Suppose that f is differentiable on \mathbb{R} and has two roots. Then, there exists an $x \in \mathbb{R}$ such that $f'(x) = 0$.

Rolle's Theorem.

Let $f : [a, b] \rightarrow \mathbb{R}$ be a function that satisfies the following properties.

- (1) f is continuous on $[a, b]$,
- (2) f is differentiable on (a, b) , and
- (3) $f(a) = f(b)$.

Then, there exists an $c \in (a, b)$ such that $f'(c) = 0$.

Theorem.

If f is differentiable, then f is continuous.