Proving Existence Nonconstructively

Proving that a solution exists doesn't always involve providing the solution itself.

Group Task

How might the given theorems be used to prove the following claim?

this is similar to the conclusion of Rolle's Thm.

> *Suppose that f is differentiable on $\mathbb{R}$ and has two roots. Then, there exists an $x \in \mathbb{R}$ such that $f'(x) = 0$.*

Rolle's Theorem.

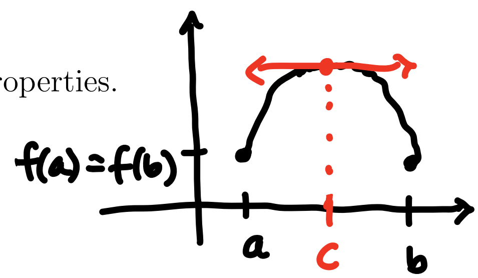
Let $f : [a, b] \to \mathbb{R}$ be a function that satisfies the following properties.

✓ (1) f is continuous on $[a, b]$,

✓ (2) f is differentiable on (a, b), and

✓ (3) $f(a) = f(b)$.

Then, there exists an $c \in (a, b)$ such that $f'(c) = 0$.



Theorem.

If f is differentiable, then f is continuous.

Proof. Assume f is differentiable on $\mathbb{R}$ and has two roots. Then, $\exists a, b \in \mathbb{R} \wedge a \neq b$ such that $f(a) = 0$ and $f(b) = 0$. Without loss of generality (WLOG), assume $a < b$. By thm, since f is diff. on $\mathbb{R}$, f is also cont on $\mathbb{R}$. In particular, f is cont. on $[a,b]$, f is diff on (a,b), and $f(a) = 0 = f(b)$. By Rolle's Thm, $\exists x \in (a,b)$ s.t. $f'(x) = 0$. Note $x \in \mathbb{R}$.