

Let  $f : A \rightarrow B$  be a function.

- We say that  $f$  is **one-to-one (1-1)** provided

$$\forall a_1, a_2 \in A, \text{ if } a_1 \neq a_2 \text{ then } f(a_1) \neq f(a_2).$$

- We say that  $f$  is **onto** provided

$$\forall b \in B, \exists a \in A \text{ such that } f(a) = b.$$

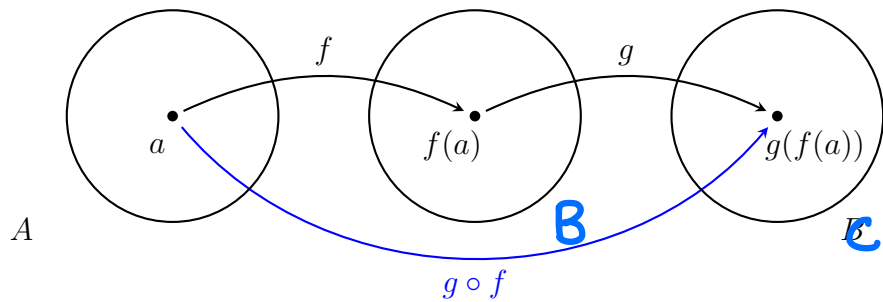
contrapositive of defn given in the HW

unequal inputs must have unequal outputs.

$f(A) = B$  (every element in  $B$  is an output of  $f$ )

### The Composition of Functions

Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions. Then the **composition**  $g \circ f : A \rightarrow C$  defined by  $g \circ f(a) = g(f(a))$  for all  $a \in A$  is also a function.



### Group Task

Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions.

- If  $g \circ f$  is 1-1, what if anything can we say about  $f$  or  $g$ ?
- If  $g \circ f$  is onto, what if anything can we say about  $f$  or  $g$ ?

①  $f$  must be 1-1.  $g$  need not be 1-1

②  $g$  must be onto but  $f$  need not be onto.

### ① Conjectures:

- ~~Both  $f$  and  $g$  are 1-1~~
- At least  $f$  must be 1-1 (correct)
- ~~At least  $g$  must be 1-1~~