$\forall b \in B,\ g(b) \in A.$

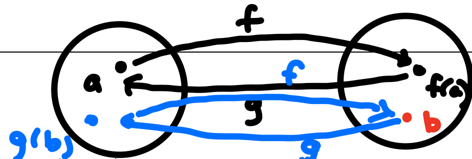
g is well-defined: (*)

Suppose that $f : A \to B$ is a function. We say that a function $g : B \to A$ is the inverse of f provided

1. $g(f(a)) = a \quad \forall a \in A$ and
2. $f(g(b)) = b \quad \forall b \in B$

f and g "undo" each other.

Reminder: pre-image of S is the set of all $a \in A$ s.t. $f(a) \in S$. denoted $f^{-1}(S)$.

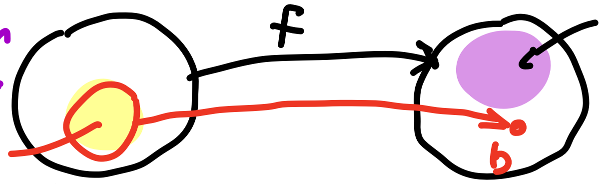


Group Task

(*) $\forall b_1, b_2 \in B$ if $b_1 = b_2$ then $g(b_1) = g(b_2)$.

1. Consider the function $f : A \to B$. For f to be invertible, what can you say must be true about the preimage of any point $b \in B$ under f, i.e. $f^{-1}(\{b\})$?

This depicts a noninvertible function



$f(A)$ (range of f)

$f^{-1}(\{b\})$ A B

i.e. $b \notin$ range(f)

what if $f^{-1}(\{b\}) = \emptyset$?

i.e. $\nexists a \in A$ s.t. $f(a) = b$.

Answer to 1: $\boxed{f^{-1}(\{b\}) \text{ must contain exactly one } a \in A.}$

The inverse of f is supposed to take b back to the $a \in A$ that $f(a)$ mapped to b.

2. What properties might f need to satisfy to guarantee that the preimage of any point $b \in B$ under f behaves as you described in question 1?

Issue: $g(b) = ??$ $\therefore g$ is not defined for b.

Ethan says: For g to be defined,

$$f(A) = B$$

range of f — domain of g

Range f = codomain of f.

Range f = domain g

① Given any $b \in B$, we need to be able to send b somewhere in A under g. i.e. $g(b)$ must be defined

* Requirement: every $b \in B$ must be in range of f. (for g to be defined)

$\Rightarrow$ range$(f) = B$

$f(A) = B$

$f^{-1}(\{b\}) \neq \emptyset$ has at