Proving Existence Constructively

Consider the statement:

There exists a positive integer x such that $x^3 - 4x^2 - x + 4 = 0$.

For each proof below, decide whether it does or does not prove this statement.

 Proof 1. Suppose that $x^3 - 4x^2 - x + 4 = 0$. Then,
 Proof 2. Let x = 1. Then,

 $x^2(x-4) - (x-4) = 0$ $(x-4)(x^2-1) = 0$ $x^3 - 4x^2 - x + 4 = (1)^3 - 4(1)^2 - (1) + 4$ = 1 - 4 - 1 + 4= 0.

 So, x = 4 or $x = \pm 1$. Since x = 1 is a positive integer, the claim is true.
 \Box