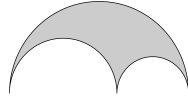




Background

Arbelos

The term "arbelos" means shoemaker's knife in Greek. An arbelos is combined with three semicircles which are shared with one of the others, all on the same side of a straight line that contains their diameters. These three circles are also tangential from each other. (Weisstein, 2020)

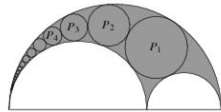


Original:

The Arbelo was first introduced in the book of Lemmas. The book of Lemmas is Thābit ibn Qurra's book attributed to Archimedes. It consists of fifteen propositions on circles. From Wikipedia.

Pappus Chain

Starting with the circle P_1 tangent to the three semicircles forming the arbelos, construct a chain of tangent circles P_i , all tangent to one of the second small interior circles and to the largest exterior one. Constructing an infinite chain of tangent circles is called the Pappus chain (left figure). (Weisstein, 2020)



Mathematician:

Pappus was an ancient Greek mathematician. He was also the last great geologist at Alexander College. His book, "Anthology of Mathematics" is of great significance to the history of mathematics.

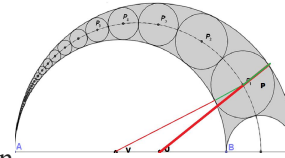
Pappus Chain

Ruisi Ma & Yimeng Liu

Properties

Center of the Circle

All the centers of the circles in the Pappus chain are located on a common ellipse, for the following reason. The sum of the distances from the n^{th} circle to the two centers U (Largest) and V (Second) of the arbelos circles equals a constant: (Weisstein, 2020)

$$P_n U + P_n V = (r_U + r_n) + (r_V - r_n) = r_U + r_V$$


Coordinate

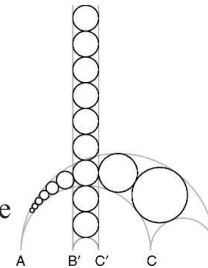
If $r = AC / AB$ then the center of the n^{th} circle is:

$$x_n = \frac{r(1+r)}{2n^2(1-r)^2 + r}$$

$$y_n = \frac{nr(1-r)}{n^2(1-r)^2 + r} \quad (\text{Weisstein, 2020})$$

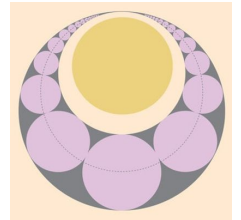
Distance from bottom to n^{th} circle's center is n time radiums:

Using inverse geometry theory, The circle with diameter AB gets mapped to the vertical line through B' , and the circle with diameter AC gets mapped to the vertical line through C' . Our Pappus chain is transformed by inversion into an infinite tower of circles. From that we could find the height of the circle: $H_3 = 3d_3 = 6r$ (Lambie-Hanson, 2017)



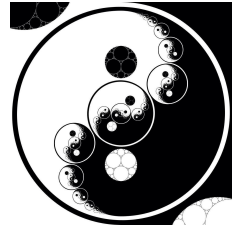
Art :

The World Necklace
Mathematically inspired Poster
(2015)



Cosmic Evolution: The Rise of Complexity in Nature by Eric J Chaisson .

@koji_glass (2019)



Further Research:

1. How kind of applications are there for the pappus chain?
2. How could the properties of the pappus chain also apply to the steiner chain?
3. How to use the pappus chain to calculate the area in geometry?

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