# FRACTALS

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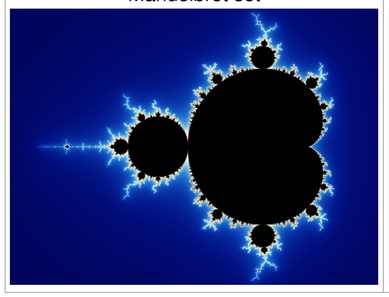
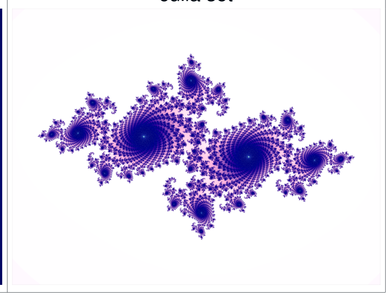
## Introduction

- A fractal is a self-similar subset of Euclidean space whose fractal dimension strictly exceeds its topological dimension[1].
- Benoît Mandelbrot described fractals as being a geometric shape that when divided into parts would be a similar replica of the original shape.[3]
- A Pure Fractal is a geometric representation of a fractal that is self-similar through infinite iterations in a recursive pattern and through infinite detail.
- All fractals make use of a recursive definition, which breaks a complex problem into smaller pieces, which are then solved the same way over and over again until reaching a specific limit. [1]

## Graphing Fractals

There are many different types of fractals, but they all stem from two sets

| Mandelbrot set | Julia set |
|---|---|
|  |  |

The **MandelBrot** set is represented by the equation

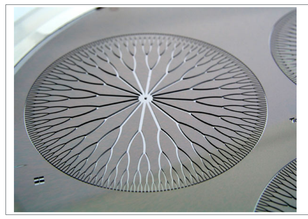
$$f_c(z) = z^2 + c$$

where $c$ is the set of complex numbers for which the function does not diverge when iterated from z = 0 [1].
All starting values of $c$ in the black parts of the Mandelbrot set cause the $z$ values to stay finite. [2]
The different colors represent how fast $z$ goes to infinity. [2]

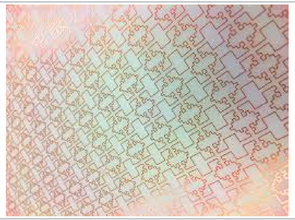
The **Julia** set is also the set of complex numbers $f_c(z) = z^2 + c$, except $z$ is initialized by the equation $z = z^2 + c$ [4].

## Fractal Applications

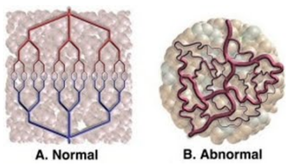
Engineers have started copying natural "fractals" to build devices such as:



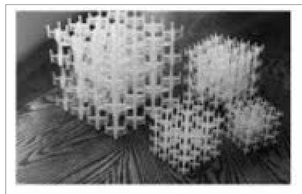
Developed by researchers at Oregon State University.
A computer chip cooling circuit that channels liquid nitrogen across the surface to keep the chip cool.
The device is etched in a fractal branching pattern to distribute the coolant.[2]



Developed by Fractenna (Fractus) in the US (Europe)
Fractal-Shaped antennas that can receive radio signals from a large range of frequencies.
Because of their fractal shapes, these antennas can be very compact.[2]




Developed by researchers at Harvard Medical school.
Researchers at Harvard Medical School and elsewhere are using fractal analysis to assess the health of blood vessels in cancerous tumors. Fractal analysis of CT scans can also quantify the health of lungs suffering from emphysema or other pulmonary illnesses.[2]



Developed at Amalgamated Research Inc (ARI)
A space-filling fractal device for high precision fluid mixing.
Useful in many industries, these devices allow fluids such as epoxy resins to be carefully and precisely blended without the need for turbulent stirring.[2]

## Fractals in Nature

Even though fractals are defined as infinite, nature's "fractals" only resemble real fractals. They do not continue indefinitely,but they do follow a similar geometric pattern as algebraic fractals. Some examples include:

**Fern**

**Romenesco Broccoli**

**Tree Branches**

## Future questions

- With all the new information about fractals, what can fractal math be used for in the future?

## References

[1] *Fractal*. Apr. 2020. URL: https://en.wikipedia.org/wiki/Fractal.
[2] *FractalPacks-EducatorsGuide.pdf*. URL: https://fractalfoundation.org/fractivities.
[3] *History of Fractals*. URL: https://sk33lz.com/create/fractals/history-fractals.
[4] *Understanding Julia and Mandelbrot Sets*. URL: https://www.karlsims.com/julia.html.