Professional Mathematicians and Pseudo-Objectification

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Research Day

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Theoretical Framework

- Piaget (1964) distinguished between two types of knowing/understanding
  - “To know an obejct, to know an event, is not simply to look at it and make a mental copy or image of it. To know an object is to *act on it*. To know is to modify, to transform the object, and *to understand the process of this transformation*, and as a consequence *to understand the way the object is constructed*" (p. 176, emphasis my own).

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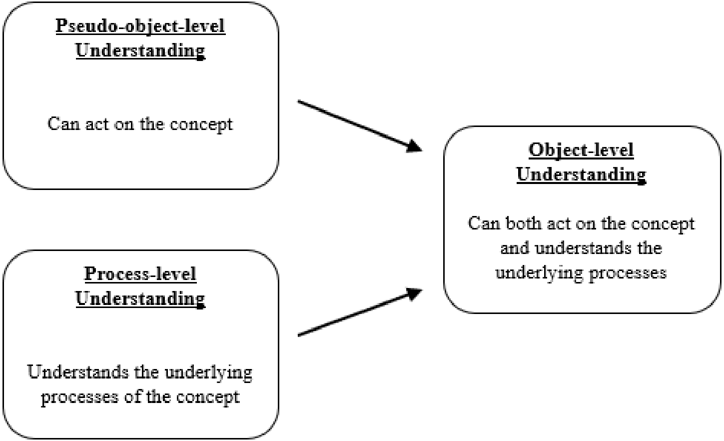
- *Process-level understanding* - When an individual has understanding of the underlying processes of the concept, but cannot act on it.
- *Pseudo-object-level understanding* - When an individual can act on the concept and use it like an object, but does not have a complete understanding of its underlying processes.
- *Object-level understanding* - When an individual understands the underlying processes of the concept, and can act on the concept.

Levels of Understanding



Mathematicians and Pseudo-Objectification

- Sfard (1991) acknowledged, “The contemporary mathematician would offer an entirely new idea in a form of a ready-made object, clearly believing that the abstract construct may be brought into being just by force of an appropriate definition. Thus the possibility must be considered that, after all, structural conception may sometimes be the first. This can certainly be true in the case of professional mathematicians - their well-trained minds can indeed be capable of manipulating abstract objects right away, without the mediation of computational processes" (pp. 22-23).

Research Questions

- (Broadly) How do professional mathematicians successfully work with and understand highly-abstract, advanced mathematical concepts?

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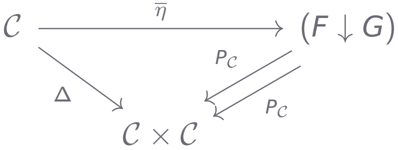
- (Broadly) How do professional mathematicians successfully work with and understand highly-abstract, advanced mathematical concepts?
- In what ways do professional mathematicians operate with highly-abstract, advanced mathematical concepts at different levels of understanding?
- What factors can influence a professional mathematician's level of understanding for a given mathematical concept?

Comma Category Task

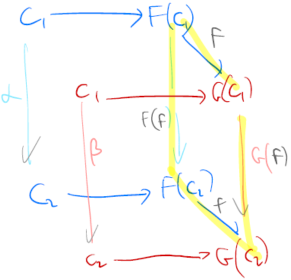
Definition: Consider categories $\mathcal{B}, \mathcal{C}, \mathcal{D}$ with functors $F : \mathcal{B} \to \mathcal{D}$ and $G : \mathcal{C} \to \mathcal{D}$. The *comma category* $(F \downarrow G)$ has as objects all triples (B, C, f) where $B \in \mathcal{B}$, $C \in \mathcal{C}$, and $f : F(B) \to G(C)$ is a morphism in $\mathcal{D}$. The morphisms from (B, C, f) to (B', C', f') are all pairs (α, β) of morphisms $\alpha : B \to B'$, $\beta : C \to C'$ such that $G(\beta) \circ f = f' \circ F(\alpha)$.

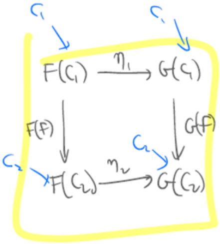
Problem: Let $F, G : \mathcal{C} \to \mathcal{D}$ be two functors. Show that a natural transformation $\eta : F \dot{\to} G$ is the same as a functor $\overline{\eta} : \mathcal{C} \to (F \downarrow G)$ such that the following diagram commutes



The Case of Aaron

- “So, I don’t really fully, I don’t holistically understand the definition [of natural transformation]...But, I don’t think I need to know much to do this, other than just looking at the diagram and showing that this is what it is."





References

- Piaget, J. (1964). Part I: Cognitive development in children: Piaget. Development and learning. *Journal of Research in Science Teaching, 2*(3), 176-186.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22*, 1-36.