38. Figure 7.5 shows the graphs of \( y = 1/x^2 \) and \( y = 1/x^3 \). We see that \( \int_1^\infty \frac{1}{x^2} \, dx \) is larger, since the area under \( 1/x^2 \) is larger than the area under \( 1/x^3 \).

![Figure 7.5]

39. (a) An antiderivative of \( F'(x) = \frac{1}{x^2} \) is \( F(x) = -\frac{1}{x} \) (since \( \frac{d}{dx} \left( -\frac{1}{x} \right) = \frac{1}{x^2} \)). So by the Fundamental Theorem we have:

\[
\int_1^b \frac{1}{x^2} \, dx = -\frac{1}{x} \bigg|_1^b = -\frac{1}{b} + 1.
\]

(b) Taking a limit, we have

\[
\lim_{b \to \infty} \left( -\frac{1}{b} + 1 \right) = 0 + 1 = 1.
\]

Since the limit is 1, we know that

\[
\lim_{b \to \infty} \int_1^b \frac{1}{x^2} \, dx = 1.
\]

So the improper integral converges to 1:

\[
\int_1^\infty \frac{1}{x^2} \, dx = 1.
\]

40. (a) Using a calculator or computer, we get

\[
\begin{align*}
\int_0^3 e^{-2t} \, dt &= 0.4988 \\
\int_0^5 e^{-2t} \, dt &= 0.4999\ldots \\
\int_2^7 e^{-2t} \, dt &= 0.4999996 \\
\int_0^{10} e^{-2t} \, dt &= 0.499999999.
\end{align*}
\]

The values of these integrals are getting closer to 0.5. A reasonable guess is that the improper integral converges to 0.5.

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### Solutions for Section 7.4

2. Since \( F(0) = 0 \), \( F(b) = \int_0^b f(t) \, dt \). For each \( b \) we determine \( F(b) \) graphically as follows:

- \( F(0) = 0 \)
- \( F(1) = F(0) + \text{Area of } 1 \times 1 \text{ rectangle} = 0 + 1 = 1 \)
- \( F(2) = F(1) + \text{Area of triangle } (\frac{1}{2} \cdot 1 \cdot 1) = 1 + 0.5 = 1.5 \)
- \( F(3) = F(2) + \text{Negative of area of triangle } = 1.5 - 0.5 = 1 \)
- \( F(4) = F(3) + \text{Negative of area of rectangle } = 1 - 1 = 0 \)
- \( F(5) = F(4) + \text{Negative of area of rectangle } = 0 - 1 = -1 \)
- \( F(6) = F(5) + \text{Negative of area of triangle } = -1 - 0.5 = -1.5 \)

The graph of \( F(t) \), for \( 0 \leq t \leq 6 \), is shown in Figure 7.7.

![Figure 7.7]
(b) Since \(-\frac{1}{2}e^{-2t}\) is an antiderivative of \(e^{-2t}\), we have

\[ \int_0^b e^{-2t} \, dt = \left[ -\frac{1}{2}e^{-2t} \right]_0^b = -\frac{1}{2}e^{-2b} - \left( -\frac{1}{2}e^0 \right) = -\frac{1}{2}e^{-2b} + \frac{1}{2}. \]

(c) Since \(e^{-2b} = 1/e^{2b}\), we have

\[ e^{2b} \to \infty \quad \text{as} \quad b \to \infty, \quad \text{so} \quad e^{-2b} = \frac{1}{e^{2b}} \to 0. \]

Therefore,

\[ \lim_{b \to \infty} \int_0^b e^{-2t} \, dt = \lim_{b \to \infty} \left( -\frac{1}{2}e^{-2b} + \frac{1}{2} \right) = 0 + \frac{1}{2} = \frac{1}{2}. \]

So the improper integral converges to \(1/2 = 0.5\):

\[ \int_0^\infty e^{-2t} \, dt = \frac{1}{2}. \]

4. (a) The value of the integral is negative since the area below the \(x\)-axis is greater than the area above the \(x\)-axis. We count boxes: the area below the \(x\)-axis includes approximately 11.5 boxes and each box has area \((2)(1) = 2\), so

\[ \int_0^5 f(x) \, dx \approx -23. \]

The area above the \(x\)-axis includes approximately 2 boxes, each of area 2, so

\[ \int_5^7 f(x) \, dx \approx 4. \]

So we have

\[ \int_0^7 f(x) \, dx = \int_0^5 f(x) \, dx + \int_5^7 f(x) \, dx \approx -23 + 4 = -19. \]

(b) By the Fundamental Theorem of Calculus, we have

\[ F'(7) - F'(0) = \int_0^7 f(x) \, dx \]

so,

\[ F(7) = F(0) + \int_0^7 f(x) \, dx = 25 + (-19) = 6. \]

5. See Figure 7.9.
9. (a) The function \( f(x) \) is increasing when \( f'(x) \) is positive, so \( f(x) \) is increasing for \( x < -2 \) or \( x > 2 \).

The function \( f(x) \) is decreasing when \( f'(x) \) is negative, so \( f(x) \) is decreasing for \(-2 < x < 2\).

Since \( f(x) \) is increasing to the left of \( x = -2 \), decreasing between \( x = -2 \) and \( x = 2 \), and increasing to the right of \( x = 2 \), the function \( f(x) \) has a local maximum at \( x = -2 \) and a local minimum at \( x = 2 \).

(b) See Figure 7.13.

![Figure 7.13](image)

![Figure 7.14](image)

10. (a) The function \( f(x) \) is increasing when \( f'(x) \) is positive, so \( f(x) \) is increasing for \(-1 < x < 3 \) or \( x > 3 \).

The function \( f(x) \) is decreasing when \( f'(x) \) is negative, so \( f(x) \) is decreasing for \( x < -1 \).

Since \( f(x) \) is decreasing to the left of \( x = -1 \) and increasing to the right of \( x = -1 \), the function has a local minimum at \( x = -1 \). Since \( f(x) \) is increasing on both sides of \( x = 3 \), it has neither a local maximum nor a local minimum at that point.

(b) See Figure 7.14.

15. For every number \( b \), the Fundamental Theorem tells us that

\[
\int_0^b f'(x) \, dx = F(b) - F(0) = F(b) - 0 = F(b).
\]

Therefore, the values of \( F(1), F(2), F(3), \) and \( F(4) \) are values of definite integrals. The definite integral is equal to the area of the regions under the graph above the \( x \)-axis minus the area of the regions below the \( x \)-axis above the graph. Let \( A_1, A_2, A_3, A_4 \) be the areas shown in Figure 7.18. The region between \( x = 0 \) and \( x = 1 \) lies above the \( x \)-axis, so \( F(1) \) is positive, and we have

\[ F(1) = \int_0^1 f'(x) \, dx = A_1. \]

The region between \( x = 0 \) and \( x = 2 \) also lies entirely above the \( x \)-axis, so \( F(2) \) is positive, and we have

\[ F(2) = \int_0^2 f'(x) \, dx = A_1 + A_2. \]

We see that \( F(2) > F(1) \). The region between \( x = 0 \) and \( x = 3 \) includes parts above and below the \( x \)-axis. We have

\[ F(3) = \int_0^3 f'(x) \, dx = (A_1 + A_2) - A_3. \]

Since the area \( A_3 \) is approximately the same as the area \( A_2 \), we have \( F(3) \approx F(1) \). Finally, we see that

\[ F(4) = \int_0^4 f'(x) \, dx = (A_1 + A_2) - (A_3 + A_4). \]

Since the area \( A_1 + A_2 \) appears to be larger than the area \( A_3 + A_4 \), we see that \( F(4) \) is positive, but smaller than the others.

The largest value is \( F(2) \) and the smallest value is \( F(4) \). None of the numbers is negative.

![Figure 7.18](image)
16. First notice that $F$ will be decreasing on the interval $0 < x < 1$ and on the interval $3 < x < 4$ and will be increasing on the interval $1 < x < 3$. The areas tell us how much the function increases or decreases. By the Fundamental Theorem, we have

$$F(1) = F(0) + \int_0^1 F'(x)\,dx = 5 + (-3) = -1.$$  
$$F(3) = F(1) + \int_1^3 F'(x)\,dx = -1 + 8 = 7.$$  
$$F(4) = F(3) + \int_3^4 F'(x)\,dx = 7 + (-2) = 5.$$  

24. See Figure 7.25.

25. See Figure 7.26.

26. See Figure 7.27.