8. Note that \( \int_a^b g(x) \, dx = g(t) \, dt \). Thus, we have
\[
\int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx = 8 + 2 = 10.
\]

9. Note that \( \int_a^b (g(x))^2 \, dx = \int_a^b (g(t))^2 \, dt \). Thus, we have
\[
\int_a^b ((f(x))^2 - (g(x))^2) \, dx = \int_a^b (f(x))^2 \, dx - \int_a^b (g(x))^2 \, dx = 12 - 3 = 9.
\]

10. We have
\[
\int_a^b (f(x))^2 \, dx - \left( \int_a^b f(x) \, dx \right)^2 = 12 - 8^2 = -52.
\]

11. Note that \( \int_a^b f(x) \, dx = \int_a^b f(x) \, dx \). Thus, we have
\[
\int_a^b cf(x) \, dx = c \int_a^b f(x) \, dx = 8c.
\]

Solutions for Section 6.1

1. (a) Counting the squares yields an estimate of 25 squares, each with area = 1, so we conclude that
\[
\int_0^5 f(x) \, dx \approx 25.
\]

(b) The average height appears to be around 5.

(c) Using the formula, we get
\[
\text{Average value} = \frac{\int_0^5 f(x) \, dx}{5 - 0} \approx \frac{25}{5} \approx 5,
\]
which is consistent with (b).

3. (a) Since \( f(x) \) is positive on the interval from 0 to 6, the integral is equal to the area under the curve. By examining the graph, we can measure and see that the area under the curve is 20 square units, so
\[
\int_0^6 f(x) \, dx = 20.
\]

(b) The average value of \( f(x) \) on the interval from 0 to 6 equals the definite integral we calculated in part (a) divided by the size of the interval. Thus
\[
\text{Average Value} = \frac{1}{6} \int_0^6 f(x) \, dx = 3 \frac{1}{3}.
\]

6. Average value = \( \frac{1}{10 - 0} \int_0^{10} e^t \, dt \approx 2202.55 \)

Solutions for Section 6.4


Between 2000 and 2001, relative increase = 2.8/21.8 = 12.8%. Between 2003 and 2004, relative increase = 3.1/30.2 = 10.3%.
3. (a) If population is decreasing at 100 per hour,

\[ \text{Population} = 4000 - 100t \text{(Number of hours)}. \]

So
\[ P = 4000 - 100t. \]

(b) If population is decreasing at 5% per hour, then it is multiplied by \((1 - 0.05) = 0.95\) for each additional hour. This means
\[ P = 4000(1 - 0.05)^t = 4000(0.95)^t. \]

The linear function \((P = 4000 - 100t)\) reaches zero first.

4. The relative growth rate is a constant 0.02 = 2%. The change in \(\ln P(t)\) is the area under the curve which is \(10(0.02) = 0.2\). So
\[
\ln P(10) - \ln P(0) = \int_0^{10} \frac{P'(t)}{P(t)} \, dt = 0.2
\]
\[
\ln \left( \frac{P(10)}{P(0)} \right) = 0.2
\]
\[
\frac{P(10)}{P(0)} = e^{0.2} \approx 1.22.
\]

The population has increased by about 22% over the 10-year period.

Another way of looking at this problem is to say that since \(P(t)\) is growing at a constant 2% rate, it is growing exponentially, so
\[ P(t) = P(0)e^{0.02t}. \]

Substituting \(t = 10\) gives the same result as before:
\[ \frac{P(10)}{P(0)} = e^{0.02(10)} = e^{0.2} \approx 1.22. \]

6. The area between the graph of the relative rate of change and the \(t\)-axis is 0 (because the areas above and below are equal). Since the change in \(\ln P(t)\) is the area under the curve
\[
\ln P(10) - \ln P(10) = \int_0^{10} \frac{P'(t)}{P(t)} \, dt = 0
\]
\[
\ln \left( \frac{P(10)}{P(0)} \right) = 0
\]
\[
\frac{P(10)}{P(0)} = e^0 = 1
\]
\[ P(10) = P(0). \]

The population at \(t = 0\) and at \(t = 10\) are equal.

17. (a) We compute the left- and right-hand sums:

Left sum = \(-0.03 + 0.03 - 0.06 \ldots + 0.03 = -0.38 \)

and

Right sum = \(0.03 - 0.06 \ldots + 0.03 + 0.02 = -0.33\),

and average these to get our best estimate
\[ \text{Average} = \frac{-0.38 + (-0.33)}{2} = -0.355. \]

We estimate that
\[ \int_{1990}^{2002} \frac{P'(t)}{P(t)} \, dt \approx -0.36. \]

(b) The integral found in part (a) is the change in \(\ln P(t)\), so we have
\[
\ln P(2002) - \ln P(1990) = \int_{1990}^{2002} \frac{P'(t)}{P(t)} \, dt = -0.36
\]
\[
\ln \left( \frac{P(2002)}{P(1990)} \right) = -0.36
\]
\[
\frac{P(2002)}{P(1990)} = e^{-0.36} = 0.70.
\]

Burglaries went down by a factor of 0.70 during this time, which means that the number of burglaries in 2002 was 0.70 times the number of burglaries in 1990. This is a decrease of about 30% during this period.