1. The rate of change of $P$ is proportional to $P$ so we have
\[ \frac{dP}{dt} = kP, \]
for some constant $k$. Since the population $P$ is increasing, the derivative $dP/dt$ must be positive. Therefore, $k$ is positive.

3. The rate at which the balance is changing is 5% times the current balance, so we have
\[ \text{Rate of change of } B = 0.05 \cdot \text{Current balance} \]
so we have
\[ \frac{dB}{dt} = 0.05B. \]

6. (a) The amount of caffeine, $A$, is decreasing at a rate of 17% times $A$, so we have
\[ \frac{dA}{dt} = -0.17A. \]
The negative sign indicates that the amount of caffeine is decreasing at a rate of 17% times $A$. Notice that the initial amount of caffeine, 100 mg, is not used in the differential equation. The differential equation tells us only how things are changing.
(b) At the start of the first hour, we have $A = 100$. Substituting this into the differential equation, we have
\[ \frac{dA}{dt} = -0.17A = -0.17(100) = -17 \text{ mg/hour}. \]
We estimate that the amount of caffeine decreases by about $(17 \text{ mg/hr}) \cdot (1 \text{ hr}) = 17 \text{ mg}$ during the first hour. This is only an estimate, however, since the derivative $dA/dt$ will not stay constant at $-17$ throughout the entire first hour.

9. The amount of morphine, $M$, is increasing at a rate of 2.5 mg/hour and is decreasing at a rate of 0.347 times $M$. We have
\[ \text{Rate of change of } M = \text{Rate in} - \text{Rate out}. \]
\[ \frac{dM}{dt} = 2.5 - 0.347M. \]
1. Since $y = t^4$, the derivative is $\frac{dy}{dt} = 4t^3$. We have

Left-side $= t \frac{dy}{dt} = t(4t^3) = 4t^4$.

Right-side $= 4y = 4t^4$.

Since the substitution $y = t^4$ makes the differential equation true, $y = t^4$ is in fact a solution.

2. (a) Since $y = x^2$, we have $y' = 2x$. Substituting these functions into our differential equation, we have

$$xy' - 2y = x(2x) - 2(x^2) = 2x^2 - 2x^2 = 0.$$Therefore, $y = x^2$ is a solution to the differential equation $xy' - 2y = 0$.

(b) For $y = x^3$, we have $y' = 3x^2$. Substituting gives:

$$xy' - 2y = x(3x^2) - 2(x^3) = 3x^3 - 2x^3 = x^3.$$Since $x^3$ does not equal 0 for all $x$, we see that $y = x^3$ is not a solution to the differential equation.

4. At $t = 0$, we know $P = 70$ and we can compute the value of $\frac{dP}{dt}$:

At $t = 0$, we have $\frac{dP}{dt} = 0.2P - 10 = 0.2(70) - 10 = 4$.

The population is increasing at a rate of 4 million fish per year. At the end of the first year, the fish population will have grown by about 4 million fish, and so we have:

At $t = 1$, we estimate $P = 70 + 4 = 74$.

We can now use this new value of $P$ to calculate $\frac{dP}{dt}$ at $t = 1$:

At $t = 1$, we have $\frac{dP}{dt} = 0.2P - 10 = 0.2(74) - 10 = 4.8$.

and so:

At $t = 2$, we estimate $P = 74 + 4.8 = 78.8$.

Continuing in this way, we obtain the values in Table 10.1.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>70</td>
<td>74</td>
<td>78.8</td>
<td>84.35</td>
</tr>
</tbody>
</table>

6. We know that at time $t = 0$, the value of $y$ is 8. Since we are told that $\frac{dy}{dt} = 0.5t$, we know that at time $t = 0$ $\frac{dy}{dt} = 0.5(0) = 0$.

As $t$ goes from 0 to 1, $y$ will increase by 0, so at $t = 1$,

$y = 8 + 0(1) = 8$.

Likewise, we get that at $t = 1$,

$\frac{dy}{dt} = 0.5(1) = 0.5$

and so at $t = 2$

$y = 8 + 0.5(1) = 8.5$.

At $t = 2$,

$\frac{dy}{dt} = 0.5(2) = 1$

then at $t = 3$:

$y = 8.5 + 1(1) = 9.5$.

At $t = 3$, $\frac{dy}{dt} = 0.5(3) = 1.5$ so that at $t = 4$, $y = 9.5 + 1.5(1) = 11$.

Thus we get the following table

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>8</td>
<td>8.5</td>
<td>9.5</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>
3. The equation given is in the form 
\[ \frac{dP}{dt} = kP. \]
Thus we know that the general solution to this equation will be 
\[ P = Ce^{kt}. \]
And in our case, with \( k = 0.02 \) and \( C = 20 \) we get 
\[ P = 20e^{0.02t}. \]

5. Rewriting we get 
\[ \frac{dy}{dx} = -\frac{1}{3}y. \]
We know that the general solution to an equation in the form 
\[ \frac{dy}{dx} = ky \]
is 
\[ y = Ce^{kx}. \]
Thus in our case we get 
\[ y = Ce^{-\frac{1}{3}x}. \]
We are told that \( y(0) = 10 \) so we get 
\[ y(x) = Ce^{-\frac{1}{3}x} \]
\[ y(0) = 10 = Ce^{0} \]
\[ C = 10 \]
Thus we get 
\[ y = 10e^{-\frac{1}{3}x}. \]

6. The equation is in the form \( dp/dq = kp \), so the general solution is the exponential function 
\[ p = Ce^{0.1q}. \]
We find \( C \) using the condition that \( p = 100 \) when \( q = 5 \). 
\[ p = Ce^{-0.1q} \]
\[ 100 = Ce^{-0.1(5)} \]
The solution is 
\[ C = \frac{100}{e^{-0.5}} = 164.87. \]
\[ p = 164.87e^{-0.1q}. \]

16. (a) Suppose \( Y(t) \) is the quantity of oil in the well at time \( t \). We know that the oil in the well decreases at a rate proportional to \( Y(t) \), so 
\[ \frac{dY}{dt} = -kY. \]
Integrating, and using the fact that initially \( Y = Y_0 = 10^6 \), we have 
\[ Y = Y_0e^{-kt} = 10^6e^{-kt}. \]
In six years, \( Y = 500,000 = 5 \cdot 10^5 \), so 
\[ 5 \cdot 10^5 = 10^6e^{-6k} \]
so 
\[ 0.5 = e^{-6k} \]
\[ k = \frac{-\ln 0.5}{-6} = 0.1155. \]
When \( Y = 600,000 = 6 \cdot 10^5 \),
Rate at which oil decreasing \[ \left| \frac{dY}{dt} \right| = kY = 0.1155(6 \cdot 10^5) = 69,300 \text{ barrels/year}. \]
(b) We solve the equation 
\[ 5 \cdot 10^4 = 10^6e^{-0.1155t} \]
\[ 0.05 = e^{-0.1155t} \]
\[ t = \frac{-\ln 0.05}{-0.1155} = 25.8 \text{ years}. \]