3. The integral \( \int_1^3 v(t) \, dt \) represents the change in position between time \( t = 1 \) and \( t = 3 \) seconds; it is measured in meters.

7. For any \( t \), consider the interval \([t, t + \Delta t]\). During this interval, oil is leaking out at an approximately constant rate of \( f(t) \) gallons/minute. Thus, the amount of oil which has leaked out during this interval can be expressed as

\[
\text{Amount of oil leaked} = \text{Rate} \times \text{Time} = f(t) \Delta t
\]

and the units of \( f(t) \Delta t \) are gallons/minute \( \times \) minutes = gallons. The total amount of oil leaked is obtained by adding all these amounts between \( t = 0 \) and \( t = 50 \). (An hour is 60 minutes.) The sum of all these infinitesimal amounts is the integral

\[
\text{Total amount of oil leaked, in gallons} = \int_0^{50} f(t) \, dt.
\]

27. From \( t = 0 \) to \( t = 5 \) the velocity is positive so the change in position is to the right. The area under the velocity graph gives the distance traveled. The region is a triangle, and so has area \( (1/2)bh = (1/2)5 \cdot 10 = 25 \). Thus the change in position is 25 cm to the right.

28. From \( t = 0 \) to \( t = 4 \) the velocity is positive so the change in position is to the right. The area under the velocity graph gives the distance traveled. The region is a triangle, and so has area \( (1/2)bh = (1/2)4 \cdot 8 = 16 \). Thus the change in position is 16 cm to the right for \( t = 0 \) to \( t = 4 \). From \( t = 4 \) to \( t = 5 \), the velocity is negative so the change in position is to the left. The distance traveled to the left is given by the area of the triangle, \((1/2)bh = (1/2)1 \cdot 2 = 1\). Thus the total change in position is \( 16 - 1 = 15 \) cm to the right.

10. (a) The area under the curve of \( P'(t) \) from 0 to \( t \) gives the change in the value of the stock. Examination of the graph suggests that this area is greatest at \( t = 5 \), so we conclude that the stock is at its highest value at the end of the 5th week. (Some may also conclude that the area is greatest at \( t = 1.5 \), making the stock most valuable in the middle of the second week. Both are valid answers.)

Since \( P'(t) < 0 \) from \( t = 1.5 \) to about \( t = 3.8 \), we know that the value of the stock decreases in this interval. This is the only interval in which the stock's value is decreasing, so the stock will reach its lowest value at the end of this interval, which is near the end of the fourth week.

(b) We know that \( P(t) - P(0) \) is the area under the curve of \( P'(t) \) from 0 to \( t \), so examination of the graph leads us to conclude that

\[
P(4) < P(3) \approx P(0) < P(1) \approx P(2) < P(5).
\]

13. We find the changes in \( f(x) \) between any two values of \( x \) by counting the area between the curve of \( f'(x) \) and the \( x \)-axis. Since \( f'(x) \) is linear throughout, this is quite easy to do. From \( x = 0 \) to \( x = 1 \), we see that \( f'(x) \) outlines a triangle of area 1/2 below the \( x \)-axis (the base is 1 and the height is 1). By the Fundamental Theorem,

\[
\int_0^1 f'(x) \, dx = f(1) - f(0),
\]

so

\[
f(0) + \int_0^1 f'(x) \, dx = f(1)
\]

\[
f(1) = 2 - 1 = \frac{3}{2}
\]

Similarly, between \( x = 1 \) and \( x = 3 \) we can see that \( f'(x) \) outlines a rectangle below the \( x \)-axis with area \(-1\), so \( f(2) = 3/2 - 1 = 1/2 \). Continuing with this procedure (note that at \( x = 4 \), \( f'(x) \) becomes positive), we get the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>3/2</td>
<td>1/2</td>
<td>-1/2</td>
<td>-1</td>
<td>-1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>