Math 2015 Test 2      Spring 2007

Instructions:
Answer the 15 multiple choice questions by circling the answers. Only the answer will count.
Answer the free response in the space provided. You must justify all work and only use methods discussed in
class.
Write your name and your student ID number at the top of the page, and sign the pledge.

I will neither give nor receive unauthorized assistance on this exam.
Signature: ________________________________

Free Response:

1.(6 pts) Evaluate $\int \cos x \sin^3 x \, dx$.

2.(7 pts) Find $\int_{1}^{2} \frac{x}{1 + x^2} \, dx$.

3.(7 pts) Solve $F'(x) = 3x^2 + \frac{1}{x}$ with $F(1) = 5$. 
4. (7 pts) Suppose that a corpse was discovered in a motel room at midnight and its temperature was 80°F. The temperature of the room is kept constant at 60°F. Two hours later the temperature of the corpse dropped to 75°F. The temperature of a corpse follows Newton’s Law of Cooling. Find the time of death. NOTE: The temperature of a corpse at the time of death is 98.6°F. You may express the time with respect to midnight.

5. (7 pts) The derivative \( f'(x) \) for a function is graphed on [0, 5]. Suppose we also know \( f(0) = 1 \). Based on this and the derivative shown, what are the \( x \) and \( y \) coordinates of the smallest and largest value that \( f(x) \) attains on [0, 5]? NOTE the placement of the \( x \)-axis.

Smallest value:

Largest value:

6. (7 pts) Evaluate \( \int_0^\infty 3e^{-2x} \, dx \).

7. (7 pts) Determine the average value of the following function in the given interval.

\[ f(t) = t^2 - 5t + 6 \text{ on } [-1, 5] \]

8. (7 pts) Find \( \int x^2 \sqrt{x^3 + 1} \, dx \)
**MULTIPLE CHOICE:** (3 pts each)

1. Find \( \int_{1}^{7} e^x \, dx \)

   (1) \(6e\) 
   (2) \(\ln(e^x) + C\) 
   (3) \(\frac{1}{e} + C\) 
   (4) \(\frac{6}{e}\)

2. Suppose we know the following:
   \(\int_{0}^{1} f(x) \, dx = 2\) 
   \(\int_{1}^{2} g(x) \, dx = -1\) 
   \(\int_{1}^{2} f(x) \, dx = 4\) 
   \(\int_{0}^{1} g(x) \, dx = 1\)

   Then \(\int_{0}^{1} (f(x) + 3g(x)) \, dx = \)

   (1) 6 
   (2) 2 
   (3) -2 
   (4) 3

3. Find \(\int_{0}^{1} (3x + 2)^5 \, dx\)

   (1) 864.5 
   (2) 900 
   (3) 461.2 
   (4) 750.8

4. In solving \(\int_{\frac{\pi}{2}}^{0} \frac{\cos(\theta)}{\sqrt{1 + \sin(\theta)}} \, d\theta\), it is helpful to make the substitution \(u = 1 + \sin(\theta)\). Then \(du = \)

   (1) \((\theta - \cos(\theta)) \, d\theta\) 
   (2) \(\cos(\theta) \, d\theta\) 
   (3) \(-\cos(\theta) \, d\theta\) 
   (4) \(\sin(\theta) \, d\theta\)

5. In solving \(\int_{0}^{\frac{\pi}{2}} \frac{\cos(\theta)}{\sqrt{1 + \sin(\theta)}} \, d\theta\), it is helpful to make the substitution \(u = 1 + \sin(\theta)\). When we complete the substitution, the lower and upper limits on the new integral (in terms of \(u\)) are

   (1) lower: -1 to upper: 2 
   (2) lower: 0 to upper: 1 
   (3) lower: 1 to upper: \(\frac{\sqrt{2}}{2}\) 
   (4) lower: 1 to upper: 2

6. Suppose we have a function \(f(x)\), with antiderivative \(F(x)\). Values for \(F(x)\) are given in the table at the right. Based on the information, what is \(\int_{1}^{2} f(x) \, dx\) ?

   (1) 1.5 
   (2) -18 
   (3) -12.5 
   (4) 12

\[\begin{array}{c|c}
 x & F(x) \\
\hline
 1.0 & 7 \\
 1.5 & 8 \\
 2.0 & -11 \\
 7.0 & 20 \\
\end{array}\]
7. Find \( \int_{1}^{2} 5x^{3} \, dx \)

1. \( \frac{2}{3} \)  
2. \( \frac{3}{2} \)  
3. \(-3\)  
4. the integral diverges

8. What is the general solution to: \( y' = -3y + 2 \)?

1. \( y = \frac{2}{3} + Ce^{-3t} \)  
2. \( y = -\frac{2}{3} + Ce^{-3t} \)  
3. \( y = \frac{2}{3} + Ce^{3t} \)  
4. \( y = Ce^{-3t} \)

9. If \( \int_{2002}^{2004} \frac{P'(x)}{P(x)} \, dx = 0.2 \), find the relative change in the population from 2002 to 2004.

1. 122.14\%  
2. 20\%  
3. 22.14\%  
4. 1.22\%

10. Calculate the area under the curve \( y = \frac{1}{x^{2}} \) from \( x = 1 \) to \( x = 2 \).

1. 2  
2. \( \frac{1}{2} \)  
3. \( -\frac{1}{2} \)  
4. ln 4

11. Choose \( u \) such that \( \int \frac{1}{x \ln |x|} \, dx = \int \frac{1}{u} \, du \).

1. \( u = x \)  
2. \( u = x \ln |x| \)  
3. \( u = \ln |x| \)  
4. \( u = \frac{1}{\ln |x|} \)

12. Find \( \int \frac{e^{t}}{1 + e^{t}} \, dt \).

1. \( \frac{e^{t}}{1 + e^{t}} + C \)  
2. \( \ln |e^{t}| + C \)  
3. \( 1 + e^{t} + C \)  
4. \( \ln(1 + e^{t}) + C \)

13. \( \int \frac{2}{t^{5}} \, dt = \)

1. \( -\frac{1}{2t^{4}} + C \)  
2. \( 2 \ln |t^{5}| + C \)  
3. \( \ln |2t^{5}| + C \)  
4. \( -\frac{10}{t^{6}} + C \)
14. Find \( \int_{1}^{2} (x+1)(x-1) \, dx \).

\[
\begin{align*}
(1) & \quad -\frac{2}{3} \\
(2) & \quad \frac{4}{3} \\
(3) & \quad -\frac{4}{3} \\
(4) & \quad 6
\end{align*}
\]

15. Find \( \int_{0}^{\pi} \sin x \cos x \, dx \).

\[
\begin{align*}
(1) & \quad 1 \\
(2) & \quad -1 \\
(3) & \quad 0 \\
(4) & \quad \pi
\end{align*}
\]

3 Bonus points: Find \( \int \frac{x^3 + x^2 + x + 1}{x} \, dx \).

1 Bonus points: State the Fundamental Theorem of Calculus, Version I OR II.