Solutions to Practice Problems for Test 2, Math 2015

1) \( \int_{-1}^{5} (3f(x) - g(x)) \, dx = 3 \int_{-1}^{5} f(x) \, dx - \int_{-1}^{5} g(x) \, dx = 3 \left( \int_{-1}^{2} f(x) \, dx + \int_{2}^{5} f(x) \, dx \right) - \left( \int_{-1}^{2} g(x) \, dx + \int_{2}^{5} g(x) \, dx \right) = 3(3 - 1) - (5 + 2) = -1 \)

There is no rule that would allow us to calculate \( \int_{-1}^{-1/2} f(x) \, dx \).

2) \( M' = 28 - 7M \) can be rewritten as \( M' = -7(M - 4) \), so the equilibrium solution is \( M = 4 \) and the general solution is \( M = 4 + C e^{-7t} \). Solving \( M(0) = 5 \) gives \( 5 = 4 + C \), or \( C = 1 \). So the solution is \( M(t) = 4 + e^{-7t} \).

3) \( F \) is increasing where \( f \) is positive, so from about \( x = 0 \) to \( x = 1.6 \). \( F \) is then decreasing for \( x \) greater than about 1.6. The largest value for \( F \) occurs at the local maximum at \( x = 1.6 \), and the smallest value is at the end of the interval (after \( F \) has been decreasing) at \( x = 4 \). A graph of the antiderivative \( F \) is shown below:

4) a) Avg Value = \( \frac{1}{6-0} \int_{0}^{6} \left( 30t - 3t^2 + 600 \right) \, dt = 218 \)

b)

5) \( \int x e^{x^2} \, dx \)

Hint: Let \( u = x^2 \). Then \( du = 2x \, dx \)

\( \int x e^{x^2} \, dx = \frac{1}{2} e^{x^2} + c \)

6) \( \int \left( 3x^4 - \frac{1}{x} + \sin(x) \right) \, dx = \frac{3}{5} x^5 \ln|x| - \cos(x) + c \)

7) \( \int \frac{2x^2}{x^2} \, dx \)

Hint: Rewrite as \( \int \left( x^2 + 2x^{-2} \right) \, dx \)

Then, \( \int \left( x^2 + 2x^{-2} \right) \, dx = 4\sqrt{x} + \frac{2x^{-1}}{3} + c \)

8) \( \int x(x+5)^6 \, dx \)

Let \( u = x+5 \). Then \( du = dx \). Also, we need to solve for \( x = u-5 \).

\( \int x(x+5)^6 \, dx = \int u^6(u-5) \, du = \int (u^7 - 5u^6) \, du = \frac{u^8}{8} - 5\frac{u^7}{7} + c \)
\[
\frac{5(x+5)^7}{8} + c
\]
9) \[
\int 3.7 \cdot (1.026)^t \, dt = \frac{3.7(1.026)^t}{\ln(1.026)} + c
\]
10) \[
\int \sqrt[3]{x} \cdot \sqrt{x^2 - 1} \, dx
\]
Let \( u = x^2 - 1 \). Then \( du = 2x \, dx \).
\[
x = \sqrt{2} \rightarrow u = 1
\]
\[
x = \sqrt{5} \rightarrow u = 4
\]
\[
\int \frac{u}{2} \cdot \frac{4}{3} \cdot u^{\frac{1}{3}} \bigg|_1^4 = \frac{1}{3} \cdot 4^\frac{2}{3} - \frac{1}{3} \cdot 1^\frac{2}{3} = \frac{1}{3} \cdot (8) - \frac{1}{3} = \frac{7}{3}
\]
11) \[
\int (e^{2z} - 4) \, dz = \frac{1}{2} e^{2z} - 4z + c
\]
12) \[
\int_1^2 \frac{8}{x^2} \, dx = -4x^{-2} \bigg|_2^1 = 3
\]
13) \[
\lim_{b \to \infty} \int_4^b 3t^{-3/2} \, dt = \lim_{b \to \infty} \left[ -6t^{-1/2} \right]_4^b = \lim_{b \to \infty} \left[ -6 \frac{1}{\sqrt{b}} + 6 \frac{1}{\sqrt{4}} \right] = 3.
\]
14) The average value of \( f(x) \) on the interval \([2,5]\) is \( \frac{1}{5-2} \int_2^5 f(x) \, dx \). We are given that \( \frac{1}{5-2} \int_2^5 f(x) \, dx = 3.5 \), so \( \int_2^5 f(x) \, dx = 3.5(3) = 10.5 \).
15) a) \( P = 170 + 30t \).
   b) \( P = 170 \cdot (1.07)^t \)