Free Response:

1.(5 pts) Write down the correct formula to use Simpson’s rule with $n = 4$ to approximate:
\[ \int_{-1}^{1} g(t) dt \quad \text{Do NOT solve.} \]

2.(7 pts) Use the Trapezoid Rule to solve $\int_{0}^{9} (2x + 1) \, dx$ with $n = 3$.

3.(4 pts) Use Euler’s Method to find $y(3)$ given $y(2) = 4$ and $y' = 3y + 1$. 
MULTIPLE CHOICE:

The 28 multiple choice questions are 3 pts each.

In questions 1 through 5 decide whether the statement is True (1) or False (2).

1. The approximation of \( \int_{1}^{4} x^2 \, dx \) using a Right Hand Sum is an underestimate.
   (1) True  (2) False

2. The approximation of \( \int_{4}^{7} (1 - 3x^3) \, dx \) using the Trapezoid Rule with \( n = 3 \) is an exact solution.
   (1) True  (2) False

3. The approximation of \( \int_{2}^{6} \sqrt{x} \, dx \) using Simpson’s Rule with \( n = 4 \) is an exact solution.
   (1) True  (2) False

4. \( \int_{a}^{b} f(x) \, dx \) always equals the area between \( f(x) \) and the \( x \)-axis on the interval \([a, b]\).
   (1) True  (2) False

5. The trapezoidal rule can only be used if there is an odd number of subintervals.
   (1) True  (2) False

Table for questions 6, 7 and 8:

<table>
<thead>
<tr>
<th>( x )</th>
<th>-6</th>
<th>-3</th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

6. Suppose \( f(x) \) is given by the Table above, and we wish to evaluate \( \int_{-6}^{6} f(x) \, dx \).
   We cannot use which of the following methods to complete the approximation:
   (1) Right-Hand Sum  (2) Simpson’s Rule  (3) Trapezoid Rule
   (4) We CAN use all of the above with the relevant data given above.

7. Suppose \( f(x) \) is given by the Table above, and we wish to evaluate \( \int_{-6}^{6} f(x) \, dx \).
   If we use a right-hand sum and all the relevant data, then we would have \( \Delta x = ? \)
   (1) 4  (2) 12/5  (3) 2  (4) 3

8. Suppose again that \( f(x) \) is given by the Table above, and we wish to evaluate \( \int_{-6}^{6} f(x) \, dx \).
   If we use a right-hand sum and all the relevant data, then the \( f(x) \) value we would use in the first subinterval would be
   (1) 8  (2) -3  (3) 5  (4) 3
9. If we have the integral of a quadratic function to approximate using Simpson’s Rule, the smallest \( n \) we can choose for our answer to be exact is

(1) 2  (2) 4  (3) 3  (4) Our answer can never be exact. Simpson’s Rule is an approximation.

10. Suppose the temperature, in degrees, of a chemical solution after \( t \) minutes is given by \( T(t) \). Then \( T'(t) \) is the rate of change of the temperature of the solution at time \( t \).

The calculation \( \int_1^5 T'(t) \, dt = -3 \) can best be interpreted as which of the following statements?

(1) The temperature of the solution has reached 3 degrees below zero after 5 minutes.

(2) The solution has cooled down by 3 degrees between 1 and 5 minutes.

(3) The average temperature of the solution between 1 and 5 minutes is 3 degrees below zero.

(4) The total rate of change of the temperature of the solution is \(-3\) degrees per minute over 1 to 5 minutes.

11. The graph of \( f(x) \) is shown below. What is \( \int_0^4 f(x) \, dx \)?

![Graph of f(x)](image_url)

(1) 3.5  (2) -2.5  (3) -5  (4) 7

12. Which of the following is the left-hand sum with 3 subintervals to estimate \( \int_{-3}^9 6' \, dt \)?

(1) \( 3 \left( 6^0 + 6^3 + 6^6 \right) \)

(2) \( 4 \left( 6^{-3} + 6^1 + 6^5 \right) \)

(3) \( 2 \left( 6^{-3} + 2 \times 4^3 + 4^9 \right) \)

(4) \( 4 \left( 6^1 + 6^5 + 6^9 \right) \)
Problems 13, 14, 15 and 16 refer to the graph below which shows the rate $r(t)$ at which water flows into or out of a reservoir. Positive rates correspond to water flowing into the reservoir. The units for $r(t)$ are hundreds of gallons per day.

\[ r(t) \text{ in hundreds of gallons per day} \]

13. At $t = 3.5$ days, is the amount of water in the reservoir increasing, decreasing, or not changing?

(1) Increasing (2) Decreasing (3) Not changing (4) Not enough information to know.

14. Over the period from $t = 0$ to $t = 4$, at which time $t$ does the reservoir contain the most water?

(1) $t = 0$ (2) $t = 1$ (3) $t = 2$ (4) $t = 4$

15. At $t = 0$ there are 7 hundred gallons in the reservoir. If we determine that $\int_{0}^{1} r(t)dt \approx -2$, how many gallons of water are in the reservoir at $t = 1$?

(1) 200 (2) 698 (3) 702 (4) 500

16. Is there MORE or LESS water at time $t = 4$ than at time $t = 0$, and how can we tell?
   You must decide both “more” or “less” and the correct reason.

(1) LESS, because $r(4) = 0$, so there is no water in the reservoir by $t = 4$.

(2) LESS, because there is more area beneath the axis than above, indicating more water has left than entered from $t = 0$ to $t = 4$.

(3) MORE, because there is more area above the axis than beneath, indicating more water has entered than left from $t = 0$ to $t = 4$.

(4) MORE, because $r(t)$ has been increasing since $t = 1$, and in fact increases more often on $t = 0$ to $t = 4$ than it decreases.

17. Which of the following is a solution to the differential equation $y' - 3y = 0$?

(1) $y = e^{3x}$ (2) $y = 3e^{x}$ (3) $y = 3x$ (4) $y = x^{3}$
18. Let \( F(x) = \int_{-2}^{x} e^{5t^2 - t} \, dt \). Then for \( x > -2 \), \( F'(x) = \)

(1) \((10x - 1) \cdot e^{5x^2 - x}\)  (2) \(e^{5x^2 - t}\)  (3) \(e^{5x^2 - x}\)  (4) \(\int_{-2}^{x} e^{5x^2 - x} \, dx\)

19. Suppose \( f(2) = 8 \) and \( \int_{-1}^{4} f'(x) \, dx = -3 \). Then \( f(4) \) is

(1) \(-3\)  (2) \(11\)  (3) \(-\frac{8}{3}\)  (4) \(5\)

20. \( \int_{0}^{8} (x - 1)^2 \, dx = \)

(1) \(\frac{58}{3}\)  (2) \(\frac{344}{3}\)  (3) \(\frac{86}{3}\)  (4) \(172\)

21. Let \( Q(t) \) be the quantity of fish in a lake after \( t \) months. Suppose 7% of the fish in the lake are caught each month, and 120 new fish enter the lake each month. Which of the following differential equations would describe the function \( Q(t) \) ?

(1) \(Q' = 120t - 0.07t\)  (2) \(Q' = 120Q + 0.07Q\)  (3) \(Q' = 120 - 0.07Q\)  (4) \(Q' = 120t - 0.07Q\)

22. Given: \( y' = 2y - t \) and \( y(2) = -1 \). Use Euler’s method to estimate \( y(3) \). \( y(3) = \)

(1) \(-4\)  (2) \(-5\)  (3) \(0\)  (4) \(-2 - t\)

23. Suppose \( C'(t) \) is the rate of change of concentration of CCl\(_4\) in a solution \( t \) hours after a reaction begins, measured in mmol/L per hour. What are the units on the result of the integral \( \int_{0}^{24} C'(t) \, dt \) ?

(1) mmol/L per hour  (2) mmol/hour  (3) mmol/L  (4) mmol \cdot hours

24. Suppose we have found that the general solution to a differential equation is \( y(t) = Ce^{2t} \). If \( y(2) = 3 \), then \( C = \)

(1) 3  (2) 2  (3) \(\frac{\ln 2}{3}\)  (4) \(3e^{-4}\)
Problems 25 and 26 refer to the following information.

Hydrocodone bitartrate is used as a cough suppressant. After the drug is fully absorbed the quantity of drug in the body decreases at a rate proportional to the amount left in the body. The half-life of hydrocodone bitartrate in the body is 3.8 hours and the dose is 10 mg.

25. Setting up an equation for the quantity, \( Q \), of hydrocodone bitartrate in the body at time, \( t \), and using the given information allows us to solve for \( k \). We find \( k \) to be

(1) \( \frac{\ln 2}{3.8} \)  
(2) \( \frac{3.8}{2} \)  
(3) \( -2 \)  
(4) \( \frac{\ln 0.5}{3.8} \)

26. Using the equation for \( Q \), how much of the 10 mg dose is still in the body after 12 hours?

(1) 0.83  
(2) 1.12  
(3) 0  
(4) 2.63

27. The fundamental theorem of calculus is

(1) \( \int_a^b f'(x)dx = f'(a) - f'(b) \)  
(2) \( \int_a^b f'(x)dx = f(b) - f(a) \)  
(3) \( \int_a^b f(x)dx = f'(b) - f'(a) \)  
(4) \( \int_a^b f(x)dx = f(b) - f(a) \)

28. Two bicyclists begin a race at time \( t = 0 \). Their forward velocities on the track are given by the two curves on the graph below. Cyclist \( A \) has velocity given by the solid curve and cyclist \( B \) has velocity given by the dashed curve.

What are the relative positions of the cyclists at \( t = 1, 1.5 \) and 2 hours?

(1) \( B \) is ahead at \( t = 1 \), they are the same position at \( t = 1.5 \) and \( A \) is ahead at \( t = 2 \).

(2) \( B \) is ahead at all of \( t = 1, 1.5 \) and 2 hours.

(3) \( B \) is ahead at \( t = 1 \) and \( t = 1.5 \), but \( A \) is ahead at \( t = 2 \).

(4) \( A \) is ahead at all of \( t = 1, 1.5 \) and 2 hours.