I. (32 pts.) Find the truth values of the following statements. Explain your reasoning.

1. $3 \geq 7$ only if $2 \leq 7$. $F \rightarrow T \equiv T$

2. $3 \geq 7$ if $2 \leq 7$. $T \rightarrow F \equiv F$

3. A square is a rectangle if and only if it has four $90^\circ$ angles or it has two $45^\circ$ angles and two $135^\circ$ angles. Omitted due to predicates instead of statements.

4. $\forall$ odd integers $n$, $\exists k \in Z$, $n = 2k + 1$. True. There is some integer $k$ for every odd integer which makes the statement true.

5. $\exists$ odd integers $n$, $\forall k \in Z$, $n = 2k + 1$. False. There is not one integer which has the form $2k + 1$ for all $k$.

6. $\exists k \in Z$, $\forall$ odd integers $n$, $n = 2k + 1$. False. There is not one $k$ which generates all $n$.

7. $\exists k \in Z$, $\exists$ odd integers $n$, $n = 2k + 1$. True. $n = 3$, $k = 1$.

8. Write the negation of #7. $\forall k \in Z$, $\forall$ odd integer $n$, $n \neq 2k + 1$

II. (10 pts.) In Stewart’s Calculus (5e) a theorem states: 
“If $g$ is continuous at $a$ and $f$ is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at $a$.”

1. Write the contrapositive.

If the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is not continuous at $a$, then $g$ is not continuous at $a$ or $f$ is not continuous at $g(a)$.

2. Write the negation.

$g$ is continuous at $a$ and $f$ is continuous at $g(a)$ and the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is not continuous at $a$. 
III. (10 pts.) Given the following information about a computer program, find the mistake using valid argument forms. (You may write the statements first using symbols.)

1. There is an undeclared variable or there is a syntax error in the first five lines.
2. If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.
3. There is not a missing semicolon.
4. There is not a misspelled variable name.

\[ \begin{align*}
&u \lor s \\
&s \rightarrow se \lor m \\
&\sim se \\
&\sim m \\
&\sim se \\
&\sim m \\
&\therefore \sim se \land \sim m \equiv \neg(se \lor m)
\end{align*} \]

\[ \begin{align*}
&s \rightarrow se \lor m \\
&\sim(se \lor m) \\
&\therefore \sim s
\end{align*} \]

\[ \begin{align*}
&u \lor s \\
&\sim s \\
&\therefore u
\end{align*} \]

IV. (16 pts.) 1. Write a truth table for \( q \rightarrow (p \rightarrow q) \).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \to q</th>
<th>q \to (p \to q)</th>
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</thead>
<tbody>
<tr>
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2. Use Theorem 1.1.1 to simplify \( q \rightarrow (p \rightarrow q) \).

\[ q \rightarrow (p \rightarrow q) \equiv q \lor (\sim p \lor q) \equiv (\sim q \lor q) \lor \sim p \lor \sim p \equiv \sim q \lor \sim p \lor q \rightarrow (p \rightarrow q) \]
Prove using the definitions given in class or show a counterexample.

1. \( \forall a, n \in \mathbb{Z}, \text{ if } a \mid n^2 \text{ and } a \leq n, \text{ then } a \mid n. \)

   False. Let \( a = 4 \) and \( n = 6. \) 4 divides 36 but 4 does not divide 6.

2. \( \forall n \in \mathbb{Z}, \ n^2 + n \) is even.

   Proof: Let \( n \) be an integer.
   
   Case 1: Suppose \( n \) is even. Then \( n = 2k \) where \( k \) is an integer.
   
   \[
   n^2 + n = (2k)^2 + 2k = 4k^2 + 2k = 2(2k^2 + k)
   \]
   
   where \( 2k^2 + k \) is an integer. Therefore \( n^2 + n \) is even.

   Case 2: Suppose \( n \) is odd. Then \( n = 2k + 1 \) where \( k \) is an integer.
   
   \[
   n^2 + n = (2k + 1)^2 + 2k + 1 = 4k^2 + 6k + 1 = 2(2k^2 + 3k + 1)
   \]
   
   where \( 2k^2 + 3k + 1 \) is an integer. Therefore \( n^2 + n \) is even.

3. \( \forall n \in \mathbb{Z}, \text{ if } n > 2 \text{ and } n \text{ is prime, then } n \text{ is odd.} \)

   Proof: Let \( n \) be a prime integer greater than 2. Suppose \( n \) is even. Then \( n = 2k \), where \( k \) is an integer.

   Since \( n \) is greater than 2, \( k \) is greater than 1, so \( n \) is a factor of two integers neither of which is 1. Therefore \( n \) is composite. This is a contradiction since \( n \) is prime. Therefore \( n \) is odd.