I. (30 pts.) 1. Prove by induction: \( \sum_{i=1}^{n} (6i \equiv 2) = n(3n + 1) \).

2. Recursively define \( a_0 = 1 \), \( a_1 = 3 \), \( a_n = 2a_{n-1} \oplus a_{n-2} \) for \( n \geq 2 \).
   
a. Calculate \( a_2 \), \( a_3 \), and \( a_4 \).

   b. Prove that \( a_n = 2n + 1 \), for every integer \( n \geq 0 \).

II. (8 pts) Use algebra of sets to show \( (A \oplus B) \oplus (A \oplus B)^c = B \oplus A \).
III. (45 pts.) Let 
\[ U = \{ x \in \mathbb{Z} | -6 \leq x \leq 6 \}, \ A = \{ x \in U \mid x \text{ is a multiple of } 5 \text{ and } x \geq 0 \}, \ B = \{ x \in U \mid x^2 - 7x + 10 = 0 \}, \text{ and } C = \{ x \in U \mid -2 \leq x \leq 3 \}. \]

1. List the elements in A, B, and C.

2. Find:
   a. \((A \cap B) \cup C\)
   b. \((A \cap B) \cap C\)
   c. \(C^C \cap (A \cap B)\)
   d. \(P(A)\)

3. Using the sets above, determine whether each of the following is true or false. Explain your reasoning.
   a. \(B \cap A\)
   b. Let \(F = \{1\}\). Then the power set of \(F\), \(P(F)\) is a partition of \(F\).
   c. For any sets D and E, if D and E are infinite sets, then \(D \cap E\) is infinite.

IV. (17 pts.) Prove each of the following using an element argument or, if false, give a counterexample.

1. \(\text{sets } A, B, \ A \cap (A \cup B) = A\)

2. \(\text{sets } A, B, C, \ A \cap (B \cup C) = (A \cup B) \cap C\)