I. (23 pts.) Klondike Al is happy only if he finds gold. Klondike Al is happy or the next claim owner finds gold. Klondike Al does not find gold. Being happy is a necessary condition for Klondike Al to keep panning for gold.

1. Write each of the above statements in symbols.

\[ k \rightarrow g \]
\[ k \vee n \]
\[ \sim g \]
\[ g \rightarrow k \]
\[ k \rightarrow g \]
\[ k \vee n \]
\[ \sim g \]
\[ \therefore k \]
\[ \therefore n \]

2. Assume each of the statements is true. Determine whether the statements below are true, false, or the truth value is inconclusive. Explain.

a. Klondike Al is happy. False. He doesn’t find gold, so the conditional \( k \rightarrow g \) can only be true if \( k \) is false.

b. The next claim owner finds gold. True. Since \( k \) is false, \( k \vee n \) can only be true if \( n \) is true.

c. Klondike Al keeps panning if and only if the next claim owner does not find gold.

True. \( F \leftrightarrow F \)

II. (14 pts.) If a parallelogram has equal angles and it also has equal sides, then it is a square.

1. Write the contrapositive of the above statement.

If a parallelogram is not a square, then it does not have equal angles or it does not have equal sides.

2. Write the negation of the above statement.

A parallelogram has equal angles and it also has equal sides, and it is not a square.

III. (21 pts.) “All rational numbers can be multiplied by some integer so that the product is an integer.”

1. Write the above statement in symbols using two quantifiers. Is this statement true or false? Explain.

\[ \forall x \in Q, \exists y \in Z, xy \in Z \]. True. If \( x = a/b \), then \( y = b \).

2. Write the negation of the statement in symbols. Is this statement true or false? Explain.

\[ \exists x \in Q, \forall y \in Z, xy \notin Z \]. False, since the original statement is true.

3. Reverse the order of your quantifiers in part 1. Is this statement true or false? Explain.

\[ \exists x \in Q, \forall y \in Z, xy \in Z \]. True. \( x = 0 \).
IV. (14 pts.) Use truth tables in each of the following.
1. Is \( \sim (p \rightarrow q) \) logically equivalent to \( p \rightarrow \sim q \)?

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Not logically equivalent since they do not have the same truth table.

2. Is \( \sim (p \rightarrow q) \) a valid argument?

\[ \therefore p \rightarrow \sim q \]

Yes it is valid since when the hypothesis is true, the conclusion is true.

V. (28 pts.) Prove the following or show a counterexample.
1. For any real number \( n \), if \( n^2 \) is irrational, then \( n \) is irrational.
   Contra-positive: For any real number \( n \), if \( n \) is rational, \( n^2 \) is rational.

Proof: Let \( n \) be a rational number. Then \( n = \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0 \).

\[ n^2 = \frac{a^2}{b^2}, a^2, b^2 \in \mathbb{Z}, b^2 \neq 0. \] Therefore \( n^2 \) is rational.

2. For integers \( a, b \), \( a \neq 0, b \neq 0 \), if \( a \mid b \) and \( b \mid a \), then \( a = b \) or \( a = -b \).

Proof:

Let \( a, b \in \mathbb{Z}, a \neq 0, b \neq 0 \). Let \( a \mid b \) and \( b \mid a \).

\[ a \mid b \Rightarrow b = ak, k \in \mathbb{Z}. \quad b \mid a \Rightarrow a = bs, s \in \mathbb{Z}. \]

\[ b = ak = (bs)k \Rightarrow 1 = sk. \] Since \( s \) and \( k \in \mathbb{Z}, s = 1 = k \)

or \( s = -1 = k. \)
Therefore \( a = b(1) = b \) or \( a = b(-1) = -b. \)

3. For integers \( m \) and \( n \), if \( (m) \mod d = a \) and \( (n) \mod d = b \), then \( (mn) \mod d = ab \).

False. Let \( m = 5, n = 2 \) and \( d = 3 \). \( 5 \mod 3 = 2. \quad 2 \mod 3 = 2. \quad (10) \mod 3 = 1 \neq 2(2) = 4. \)