Practice Test 3

1. Definitions: Define:
   a. function  b. surjective  c. onto  d. countable  e. range  f. bijective
   g. one to one correspondence  h. uncountable  i. injective  j. one to one  k. countably
   infinite  l. cardinality  m. relation  n. reflexive  o. symmetric  p. transitive

2. Let A and B be finite sets, let \( f: A \rightarrow B \) be a function.
   a. If \( f \) is one to one, what do you know about \( |A| \) and \( |B| \)?
   b. If \( f \) is onto, what do you know about \( |A| \) and \( |B| \)?

3. Let A, B, C, and D be sets, let f, g, and h be functions such that \( f: A \rightarrow B, g: B \rightarrow C, \) and \( h: C \rightarrow D \).
   a. If \( g \circ f \) is one to one, must \( f \) be one to one? Must \( g \) be one to one? Prove or give counterexamples.
   b. If \( g \circ f \) is onto, must \( f \) be onto? Must \( g \) be onto? Prove or give counterexamples.
   c. If \( g \circ f \) is a bijection, must \( f \) be a bijection? Must \( g \) be a bijection? Prove or give counterexamples.
   d. If \( f, g, \) and \( h \) are all bijections, must \( h \circ g \circ f \) be a bijection? Prove or give counterexamples.

4. Let \( F: \mathbb{Z} \rightarrow \mathbb{Z} \) by \( F[x] = e^x \). Is \( F \) one to one? Is \( F \) onto? Prove and/or give counterexamples.

5. Let \( D: \{\text{set of all polynomials in } x\} \rightarrow \{\text{set of all polynomials in } x\} \) by \( D[p(x)] = p'(x) \) (the derivative of \( p \)). Is \( D \) one to one? Is \( D \) onto? Prove and/or give counterexamples.

6. Let A be a set with \( n \) elements. Prove \( |\mathcal{P}(A)| = 2^n \) (I would suggest looking at the similar proof done in class).

7. Define \( f: \{0, 1\}^3 \rightarrow \{0, 1\} \) by \( f[x, y, z] = (3x + 4y - 5z) \mod 2 \). Create an input/output table for \( f \).

8. Given 23 integers, must there be two integers with the same remainder when divided by 10? Must there be 3? Must there be 4? Must there be 5?

9. Given 42 people, what is the maximum number of people you can be absolutely sure of who share the same birth month?

10. Given 28 people in a class, must there be at least 6 people who flunk? Must there be at least 7 that share the same grade?

11. Prove \( \mathbb{Z} \) is countable. (Give everything that is needed.)

12. Prove \( 3\mathbb{Z} \) is countable. (Give everything that is needed.)

13. Prove \( [0, 1] \) is uncountable.

14. Prove if both \( f: A \rightarrow B \) and \( g: B \rightarrow C \) are bijections, that \( g \circ f \) is one to one and onto.

15. Let \( A =\{0, 1, 2, 3\} \) and let \( B =\{2, 3, 4\} \), and denote \( aRb \iff ab \geq 4 \). Find \( R \) (as a subset of \( A \times B \)) Find the inverse relation \( R^{-1} \).

16. Let \( A =\{0, 1, 2, 3, 4\} \) Define a binary relation \( R \) on \( A \) that is:
   a. Reflexive but not symmetric
b. Symmetric and Transitive but not Reflexive

c. Transitive but not Symmetric and not Reflexive

d. Reflexive, Symmetric, and Transitive.

17. Let \( A = \{a, b, c, d\} \) and let \( R = \{(a,b), (b,c), (b, d), (d, a), (d, b)\} \). Find the transitive closure of \( R \), \( R^+ \).

18. Let \( M \) be a binary operation on \( G \), defined by \( x \ M \ y \iff xy \geq 0 \). Is \( M \):
   a. Reflexive?
   b. Symmetric?
   c. Transitive?