I. Consider the following statement:
   *If x and y are odd integers, then x + y is an even integer.*

1. Prove the statement directly.
2. Prove the statement by contrapositive.
3. Prove the statement by contradiction.

II. Use the Quotient-Remainder Theorem to prove that for any integer n, \( 3 \mid n(n+2)(n-2) \).

III. Prove or give a counterexample.
1. \( \forall a, b \in \mathbb{Z}^+, \text{ if } a \mid b \text{ and } a \mid (b + 2), \text{ then } a = 1 \text{ or } a = 2. \)
2. \( \forall a, b \in \mathbb{Z}, \text{ if } a < b, \text{ then } a^2 < b^2. \)
3. \( \forall a, b \in \mathbb{Z}, \text{ if } a < b, \text{ then } a^2 < b^2. \)
Answers:

I. 1. Proof: Let x and y be odd integers. Then \( x = 2m + 1 \) and \( y = 2n + 1 \), where \( m \) and \( n \) are integers. \( x + y = 2m + 1 + 2n + 1 = 2m + 2n + 2 = 2(m + n + 1) \) where \( m + n + 1 \) is an integer. Therefore \( x + y \) is an even integer.

2. If \( x + y \) is an odd integer, then \( x \) or \( y \) is an even integer.
Proof: Let \( x + y \) be an odd integer. \( x + y = 2k + 1 \) where \( k \) is an integer. \( x = 2k + 1 - y \). If \( y \) is even, the conclusion is proved. Suppose \( y \) is odd. Then \( y = 2r + 1 \) where \( k \) is an integer. \( x = 2k + 1 - (2r + 1) = 2k + 1 - 2r - 1 = 2k - 2r = 2(k - r) \) where \( k - r \) is an integer. Then \( x \) is even. Therefore either \( x \) or \( y \) is even.

3. Proof: Let \( x \) and \( y \) be odd integers. Suppose \( x + y \) is odd. Then \( x + y = 2k + 1 \) where \( k \) is an integer. Because \( x \) and \( y \) are odd integers, \( x = 2m + 1 \) and \( y = 2n + 1 \), where \( m \) and \( n \) are integers. \( x + y = 2m + 1 + 2n + 1 = 2m + 2n + 2 = 2(m + n + 1) \) where \( m + n + 1 \) is an integer. Therefore \( x + y \) is an even integer. Contradiction to \( x + y \) being odd. Therefore \( x + y \) is even.

II. Proof: Let \( n \) be an integer. Then \( n = 3q, 3q + 1 \) or \( 3q + 2 \) where \( q \) is an integer by the Quotient-Remainder Theorem.

Case 1. \( n = 3q \): \( n(n + 2)(n - 2) = (3q)(3q + 2)(3q - 2) = 3[q(q + 2)(3q - 2)], \) So \( 3n \).

Case 2. \( n = 3q + 1 \): \( n(n + 2)(n - 2) = (3q+1)(3q + 3)(3q - 1) = 3[(q+1)(3q + 1)(3q - 1)], \) So \( 3n \).

Case 3: \( n = 3q + 2 \): \( n(n + 2)(n - 2) = (3q+2)(3q + 4)(3q) = 3[(q)(3q + 2)(3q +4)], \) So \( 3n \).

III. 1. True
Proof: Let \( a, b \) be positive integers, \( ab \) and \( a(b+2) \). \( b = at \) and \( b + 2 = as \) where \( t \) and \( s \) are integers. \( b = at = as - 2 \), so \( 2 = as - at = a(s - t) \) where \( s - t \) is an integer. Therefore \( a \) or \( a = 2 \).

2. True
Let \( a = 1 \) and \( b = 2 \). \( 1 < 2 \) and \( (1)^2 < (2)^2 \)

3. False
Let \( a = -4 \) and \( b = -3 \). \( -4 < -3 \), but \( (-4)^2 \) is not \( (-3)^2 \)