Chapter 2 Review

I. Let $P(x,y)$ be the statement “$x$ has taken class $y$”, $A$ is the set of students in this class, and $B$ is the set of all computer science courses at Tech. Express each of the following in English.

1. $\forall x \in A, \exists y \in B, \ P(x,y)$.
2. $\exists y \in B, \forall x \in A, \ P(x,y)$.
3. $\exists x \in A, \forall y \in B, \ P(x,y)$.
4. $\exists y \in B, \exists x \in A, \ P(x,y)$.
5. $\forall x \in A, \forall y \in B, \ P(x,y)$.

II. Determine whether each is true or false. Explain your reasoning.

1. $\forall x \in R, \exists y \in R, \ xy = x$.
2. $\forall x \in R, \exists y \in R, \ xy = x$.
3. $\forall x \in R, \forall y \in R, \ xy = x$.
4. $\exists x \in R, \forall y \in R, \ xy = x$.

III. a. Write using $\land$, $\lor$ or $\neg$.

   b. Write the negation of the statement.

   1. The product of any two nonzero integers is greater than zero.
   2. Every action has an equal and opposite reaction.

IV. The statement “No one will be admitted unless over 18 years of age” has the structure

   $\forall x \in S, \ p \implies q$.

   1. What is $S$? What is $p$? What is $q$? Write $p$ and $q$ in complete sentences.
   2. Write the negation of the statement.

V. From the following premises, determine whether the conclusions numbered 1 – 4 are valid or invalid. Show all of your work.

   No kitten that loves fish is unteachable.
   Kittens with whiskers always love fish.
   No teachable kitten has green eyes.
   My cat Puff does not have whiskers.
   My cat Muff has green eyes.

   1. Puff does not like fish.
   2. Muff does not like fish.
   3. Puff is unteachable.
   4. Muff is unteachable.

VI. An argument is valid if, and only if, whenever all premises are true, then the conclusion is true. Suppose a valid argument has a false conclusion. Which of the following statements is true of the argument?

   1. All premises are false.  2. Some premises are false.  3. All premises are true.
Answers:

I. 1. Every student in this class has taken at least one computer science course at Tech.
2. All computer science courses at Tech have been taken by at least one student in this class.
3. Some students in this class have taken all of the computer science courses at Tech.
4. There are some computer science courses at Tech that all of the students in this class have taken.
5. Some students in this class have taken some computer science courses at Tech.

II. 1. True. \( y = 1 \)
2. True. \( x = 0 \).
3. True. \( y = 1 \)
4. False. Not all values of \( y \) can be multiplied to all values of \( x \) to get \( x \).

III. a. 1. \[ x, y \in \mathbb{Z} \setminus \{0\}, \ xy > 0. \] 2. \( x \) actions \( x \), \( y \) reactions \( y \), \( x \) is equal and opposite to \( y \).
   b. 1. \[ x, y \in \mathbb{Z} \setminus \{0\}, \ xy \neq 0. \] 2. \( x \) action \( x \), \( y \) reactions \( y \), \( x \) is not equal or not opposite to \( y \).

IV. 1. \( S \) is the set of all people. \( p: x \) will be admitted. \( q: x \) is over 18 years of age.
   2. \( x \) will be admitted and \( x \) is not over 18 years of age.

V. 1. invalid
   2. valid
   3. invalid
   4. valid

VI. 2