Math 2534 – Chapter 1

1. Convert the statements to symbolic form and then use valid argument forms to deduce the conclusion. Show all steps.

If I am rested, then I think better and I am alert.
If I think better, then I work homework problems well.
If I am alert and I work homework problems well, then I want to do more problems or I want class to be longer.
I don’t want class to be longer.
I am rested.
| I want to do more problems.

2. Write the following argument in symbols.
   It’s not raining.
   Therefore, if it’s raining, then it’s foggy.
   a. Use truth tables to determine whether this is a valid argument.
   b. If valid, use valid argument forms and/or logical equivalences to deduce the conclusion from the premise. (Hint: Convert "Æ" to "⁄".)

3. Use the properties in Theorem 1.1.1 to verify the following logical equivalence. Show all steps. (Hint: Convert "Æ" to "⁄".)
   \[(p \land q) \land \neg q \land (r \land q) \equiv (q \land p)\]

4. Write the negation, contrapositive, converse, and inverse of the following statement. Use DeMorgan’s laws if necessary. Write the statements in words.
   If Joe is my father, then Randy is my uncle, but Bill is my cousin.

5. Let \(p\), \(q\), and \(r\) be statements for which \(p \land (q \lor r)\) is false and \(r\) is false. What are the truth values for:
   a. \(p \lor q\)
   b. \(~p \land q\)
   c. \(q \lor p\)
   d. \(~q \lor ~p\)

6. True or false? Explain.
   a. An equivalent way to express the inverse of “\(p\) is sufficient for \(q\)” is “\(~p\) is necessary for ~\(q\)”.
   b. An equivalent way to express the converse of “\(p\) is necessary for \(q\)” is “\(p\) only if \(q\)”.
Answers

1. symbols

\[ r \to t \to a \]
\[ t \to h \]
\[ (a \to h) \to (p, c) \]
\[ \sim c \]
\[ r \]
\[ \sim p \]

deduction
\[ r \to t \to a \]
\[ r \]
\[ \sim t \to a \]
\[ t \to a \]
\[ \sim t \to a \]
\[ t \to h \]
\[ t \]
\[ \sim h \]
\[ a \]
\[ \sim a \to h \]
\[ a \to h \]
\[ \sim a \to h \]
\[ (a \to h) \to (p, c) \]
\[ a \to h \]
\[ \sim p, c \]
\[ p, c \]
\[ \sim c \]
\[ \sim p \]

2. \( \sim r \)

\( \square r \to f \)

a. Valid

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \sim r )</th>
<th>( \to r \to f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<td>T, T</td>
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<td>F</td>
<td>F</td>
<td>T, T</td>
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</tbody>
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b. \( \sim r \)  \( \sim \sim r \) \( f \equiv r \to f \)
3

\[(p \land q) \equiv (\sim q \land (r \land q))\]
\[= (\sim p \land q) \equiv (\sim q \land (r \land q))\]
\[= (\sim p \land q) \equiv \sim q\]
\[= (\sim p \land q) \equiv (\sim q \land q)\]
\[= (\sim p \land q) \equiv \sim q\]
\[= \sim (p \land q) \equiv (q \land p)\]

4. Negation: Joe is my father and Randy isn’t my uncle or Bill isn’t my cousin.
Contrapositive: If Randy isn’t my uncle or Bill isn’t my cousin, then Joe isn’t my father.
Converse: If Randy is my uncle and Bill is my cousin, then Joe is my father.
Inverse: If Joe isn’t my father, then Randy isn’t my uncle or Bill isn’t my cousin.

5. a. F  
   b. T  
   c. F  
   d. T  

6. a. F  The inverse of “p is sufficient for q” is \(\sim p \land \sim q\).
“\(\sim p\) is necessary for \(\sim q\)” is \(\sim p \land \sim q \equiv p \land q\). They are not the same.

b. The converse of “p is necessary for q” is \(p \land q\).
“p only if q” is \(p \land q\). They are the same.