Functions

A function from a set $X$ to a set $Y$, $f: X \rightarrow Y$, is a relationship between the elements of the two sets such that each element in $X$ is related to a unique element in $Y$. 
X is the domain of f
Y is the co-domain of f

We say that for every \( x \in X, \ x \xrightarrow{\ f\ } y, \ y \in Y \)
\( y = f(x) \)

The range of \( f = \{ y \in Y \mid y = f(x) \text{ for some } x \in X \} \)

The inverse image of \( y = \{ x \in X \mid f(x) = y \} \).

For both examples of functions shown, Domain =

Co-Domain =

Range\(_1 \) =  Inverse image of \( y_2 = \)

Range\(_2 \) =  Inverse image of \( y_1 = \)

Inverse image of \( y_2 = \)  Inverse image of \( y_3 = \)

ex. \( y = x^2, \ f : R \rightarrow R \)
Definition: Suppose $f$ and $g$ are two functions from $X$ to $Y$. 

$$f = g \quad f(x) = g(x) \quad \forall x \in X$$

ex.

$$f(x) = 2x \quad g(x) = \frac{2x^3 + 2x}{x^2 + 1}$$

Functions

$$f : R \rightarrow R$$

ex.
The Hamming Distance Function gives a measure of the difference between two strings of zeros and ones that have the same length. Let $\mathbb{B} = \{0, 1\}$. $\mathbb{B}^n$ is the set of all strings of zeros and ones of length $n$. Define

$$H : \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{Z}^{\text{nonneg}}, \quad H(s, t) = \text{the number of positions in which } s \text{ and } t \text{ have different values.}$$

$n = 8$

$H(10111001, 00011101) =$
Do:

1. $f : X \rightarrow Y$ is defined by $\{(a, 1), (b, \sqrt{2}), (c, \sqrt{3}), (d, 2)\}$
   Find the domain, range, and inverse image of $\sqrt{2}$.

2. Let $P$ be the set of all U.S. presidents. For $p \in P$, $f(p)$ = the successor of $p$. Is $f$ a function?

3. Let $X =$ the set of SSNs of students in this class. Define $f$ such that $f(SSN) = SSN \mod 5$.
   Find $f(234567890)$ and $f(123456789)$. 