Possibility Trees and the Multiplication Rule

ex. How many ways can a 3 digit number made up of digits from the set \{0, 1, 2, 3\} be represented assuming that repeated digits are not allowed?

How many odd non-repeating 3 digit numbers are there?
Multiplication Rule

If an operation consists of k steps and the first step can be performed in $n_1$ ways, the second step can be performed in $n_2$ ways, ..., the $k^{th}$ step in $n_k$ ways, then the entire operation can be performed in $n_1n_2n_3...n_k$ ways.

Consider a club with 5 members {Andy, Bill, Cathy, David, Evelyn}

ex. In how many ways can this group select a president and secretary if no one holds more than one office?

ex. In how many ways can a president and secretary be selected if the president must be female?

ex. In how many ways can a president and secretary be selected if the two officers must not be the same sex?
ex. In the first example with 3 digit numbers, how many even numbers are there?

The Addition Rule

Suppose a finite set $A$ consists of $k$ distinct mutually disjoint subsets $A_1, A_2, \ldots, A_k$. \( (A = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k) \) Then the number of elements in $A$, $n(A)$, = $n(A_1) + n(A_2) + \ldots + n(A_k)$.

ex. How many strings of hexadecimal digits consist of from one through four digits? (Hexadecimal numbers are constructed using 0, 1,..., 9, A,..., F) The digits can repeat.
The Difference Rule

If $A$ is a finite set and $B \subseteq A$, then $n(A \setminus B) = n(A) - n(B)$.

ex. How many hexadecimal numbers contain repeated digits.

Do: 1. A combination lock requires three selections of numbers each from 1 through 20.
   a. How many different combinations are possible?
   b. Suppose the locks are constructed in such a way that no number can be used twice. How many different combinations are possible?

2. a. How many arrangements of no more than three letters can be formed using the letters of the word NETWORK (with no repetitions allowed).

   b. How many non repeating arrangements of three letters can be made if the letters O and R must be next to each other