Multiple Quantifiers

“There is a person supervising every detail of the production process.”

Does this mean:
• There is a single person who supervises all of the details of the production process.
• For any particular production detail, there is a person who supervises that detail. There might be different supervisors for different details.

There is ambiguity in everyday language.

ex. You can fool all of the people some of the time.

Switch quantifiers:
1. **Circles**($x$), there is a figure below it in the same column.
2. **Squares**($x$), there is a figure above it in the same column.

3. **Circle**($x$), all triangles are below it.
4. **Circle**($x$), all triangles are to the right of it.
5. **Circle**($x$), **Squares**($y$), $x$ and $y$ are the same color.
ex.
\[
\lim_{x \to a} f(x) = L \quad \exists d > 0, \quad \forall x, 0 < |x - a| < d \quad |f(x) - L| < \varepsilon
\]

Are the following T or F?

1. \(x \in \mathbb{R}, \quad y \in \mathbb{R}, \quad x + y = x\)

2. \(x \in \mathbb{R}, \quad y \in \mathbb{R}, \quad x + y = x\)

3. \(x \in \mathbb{R}, \quad y \in \mathbb{R}, \quad x + y = x\)

4. \(x \in \mathbb{R}, \quad y \in \mathbb{R}, \quad x + y = x\)
1. Write using quantifiers, then
2. Interchange the symbols \( \sqcap \) and \( \sqcup \).
3. Is either statement true?

Every real number can be multiplied to some other real number so that the product of the two numbers is zero.
Do. 1. Write using quantifiers, then
2. Interchange the symbols $\cap$ and $\supset$.
3. Is either statement true?

I. Every rational number can be written as a quotient of some integers a and b.

II. There are some real numbers x whose product with any real number y is 1.
Statements with multiple quantifiers have the form:

“Quantifiers, Predicate”

ex.

\[ \forall x \forall A, \forall y \forall B, \ P(x) \]

You can fool all of the people some of the time.

\[ p: \quad \forall x, \forall y, \ P(x) \]

\[ \sim p: \quad \forall x, \forall y, \sim P(x) \]

\[ q: \quad \forall x, \forall y, \ P(x) \]

\[ \sim q: \quad \forall x, \forall y, \sim P(x) \]
ex. Any even integer equals twice some other integer.

Be sure to use positive symbols $\square$ and $\sqcap$.

ex.  $r$:  $\square x \sqcap R, \sqcap y \sqcap R, \ P(x)$
       “All $x$ have property $P(x)$.”

$\sim r$:  $\square x \sqcap R, \sqcap y \sqcap R, \ \sim P(x)$
         “Some $x$ do not have property $P(x)$.”

Do NOT use:
ex. \[ x \in R, \ y \in R, \ xy = 2 \]

ex. Negate the Tarski’s statements below.

- \[ \neg \text{Squares}(x), \text{there is a figure above it in the same column.} \]

- \[ \neg \text{Circle}(x), \text{all triangles are below it.} \]
Do:  1. Write with quantifiers.
    2. Write the negation.

I. Any real number is smaller than some other real number.

II. You can’t fool all of the people all of the time (tricky).