1. (10 pts.) For the integral \( \int_{\frac{1}{3}}^{\frac{1}{3}} \int_{x^2}^{3x^2} dy \) sketch the region of integration and then write an integral with the order of integration reversed. Do not evaluate this integral.

\[
\int_{\frac{1}{3}}^{\frac{1}{3}} \int_{x^2}^{3x^2} dy \; dx
\]

2. (15 pts.) Rewrite \( \int_{0}^{2} \int_{2-x}^{\sqrt{4-x^2}} y - x \; dy \; dx \) in polar coordinates. Evaluate this integral.

\[
\int_{\frac{\pi}{2}}^{\pi} \int_{0}^{2} (r \sin \theta - r \cos \theta) r \; dr \; d\theta = \int_{0}^{\frac{\pi}{2}} \int_{0}^{2} (\sin \theta - \cos \theta) r^2 \; dr \; d\theta = \int_{0}^{\frac{\pi}{2}} \left[ \frac{3}{4} r^3 \right]_{0}^{2} (\sin \theta - \cos \theta) \; d\theta =
\]

\[
\int_{0}^{\frac{\pi}{2}} \frac{8}{3} (\sin \theta - \cos \theta) + \frac{8}{3} (\sin \theta - \cos \theta) \frac{1}{u^3} \; du, \ u = \cos \theta + \sin \theta, \ du = -\sin \theta + \cos \theta \; d\theta
\]

\[
= \frac{8}{3} (-\cos \theta - \sin \theta) - \frac{4}{3} u^{-2} = \frac{8}{3} (-\cos \theta - \sin \theta) - \frac{4}{3} (\cos \theta + \sin \theta)^2 \bigg|_{\frac{\pi}{2}}^{0} = 0
\]
II. Set up only a triple integral to find the volume of the solid bounded by the cylinder $z = y^2$ and the plane $x + z = 1$ in the first octant using a trace or projection in the following planes:

1. (12 pts.) $yz$ plane. Graph the 3D region and also graph the 2D region you are using for the outer limits.

$$\int_0^1 \int_{y^2}^{1} \int_0^{1-y^2} \, dx \, dy \, dz$$ or $$\int_0^1 \int_0^{\sqrt{1-x}} \int_0^y \, dy \, dz \, dx$$

2. (9 pts.) $xz$ plane. Also graph the 2D region you are using for the outer limits.

$$\int_0^1 \int_0^{\sqrt{1-x}} \int_0^{\sqrt{1-z-x}} \, dy \, dz \, dx$$ or $$\int_0^1 \int_0^{1-x} \int_0^\sqrt{1-z} \, dy \, dx$$

3. (9 pts.) $xy$ plane. Also graph the 2D region you are using for the outer limits.

$$\int_0^1 \int_{\sqrt{1-x}}^{1-x} \int_0^1 \, dz \, dx$$ or $$\int_0^1 \int_0^{1-y} \int_0^{1-x} \, dz \, dx$$
III. 1. (15 pts.) Set up an integral in cylindrical coordinates to find the volume of the smaller solid bounded by the cylinder \( x^2 + y^2 = 1 \), the cone \( x^2 + y^2 - z^2 = 0 \), and the plane \( z = \sqrt{3} \). 

\[ \int_0^{\sqrt{3}} \int_1^{\sqrt{3}/r} r dz dr d\theta \]

2. (12 pts.) For the same solid, set up an integral in spherical coordinates.

\[ \int_0^{2\pi} \int_0^{\pi/2} \int_0^{\sqrt{3}/\sin \phi} \rho^2 \sin \phi d\rho d\phi d\theta \]

IV. (12 pts.) 1. Find the mass of the solid in the first octant bounded by \( x = 2 \) and \( y + z = 1 \). Its density is given by \( \rho(x,y,z) = 2x \).

\[ \int_0^2 \int_0^{1-y} 2x \ dz dy dx = \int_0^2 \int_0^{1-y} 2x \ dz \bigg|_0^{1-y} \ dy dx = \int_0^2 \int_0^{1-y} 2x(1-y) \ dy dx = \int_0^2 2x \ dy \bigg|_0^{1-y} \ dx = \int_0^2 2x \ dy \bigg|_0^{1-y} \ dx = \int_0^2 2x \ dy \bigg|_0^{1-y} \ dx = \frac{1}{2} x^2 \bigg|_0^2 = 2 \]
2. (6 pts.) Set up only integrals to find the y coordinate of the center of mass of the solid in (1) above.

\[ \bar{y} = \frac{\int_0^1 \int_0^{1-y} 2xy \, dz \, dy \, dx}{\int_0^1 \int_0^{1-y} 2x \, dz \, dy \, dx} \]

**Extra Credit** (5 pts.): Evaluate \( \int_0^{\pi/4} \int_y^\sqrt{1-y^2} \sqrt{x^2 + y^2} \, dx \, dy \)

\[
\int_0^{\pi/4} \int_0^1 r^2 \, dr \, d\theta = \int_0^{\pi/4} \left[ \frac{r^3}{3} \right]_0^1 \, d\theta = \int_0^{\pi/4} \frac{1}{3} \, d\theta = \frac{1}{3} \left[ \theta \right]_0^{\pi/4} = \frac{\pi}{12}
\]