I. 1. (12 pts.) Graph the surface listed above each set of axes.

\[ x^2 - y^2 - z^2 = 1 \]

\[ x^2 - y^2 = 1 \quad \text{cylinder} \]

2. (7 pts.) On the axes below graph and label the level curves for \( z = -1, 0, \) and \( 1 \) for the function listed. (Hint: Solve the equation for \( z \)).

\[ x^2 - y^2 - z^2 = 1 \]

3. (5 pts.) Find the gradient vector of \( z = f(x,y) = \sqrt{x^2 - y^2} - 1 \) at the point \( (2, \sqrt{2}) \). Draw and label it on your 2D graph of the level curves.

\[ \nabla f = \left( \frac{2x}{2\sqrt{x^2 - y^2} - 1}, \frac{-2y}{2\sqrt{x^2 - y^2} - 1} \right) = \langle 2, -\sqrt{2} \rangle \text{ at the point.} \]

4. (5 pts.) What is the maximum rate of change of \( z = f(x,y) = \sqrt{x^2 - y^2} - 1 \) at \( (2, \sqrt{2}) \)?

\[ \sqrt{(2)^2 + (-\sqrt{2})^2} = \sqrt{6} \]
5. (7 pts.) Find the derivative of the function \( z = f(x,y) = \sqrt{x^2 - y^2} - 1 \) at the point \( (2, \sqrt{2}) \) in the direction \( \vec{i} - \sqrt{2} \vec{j} \)

\[
\langle 2, -\sqrt{2} \rangle \cdot \frac{\langle 1, -\sqrt{2} \rangle}{\sqrt{3}} = \frac{4}{\sqrt{3}}
\]

6. (7 pts.) Find the equation of the tangent plane of the function \( z = f(x,y) = \sqrt{x^2 - y^2} - 1 \) at the point \( (2, \sqrt{2}) \).

\[
\nabla f(x,y,z) = \langle 2x, -2y, -2z \rangle
\]

\[
4x - 2\sqrt{2}y - 2z = \langle 4, -2\sqrt{2}, -2 \rangle \cdot \langle 2, \sqrt{2}, 1 \rangle
\]

\[
4x - 2\sqrt{2}y - 2z = 2 \quad \text{or} \quad 2x - \sqrt{2}y - z = 1
\]

II. (7 pts.) Find the limit if it exists, or show the limit doesn’t exist.

\[
\lim_{(x,y) \to (0,0)} \frac{3x^2y}{4x^2 + y^2} = 0
\]

\[
\lim_{r \to 0} \frac{3r^3 \cos^2 \theta \sin \theta}{r^2 (4 \cos^2 \theta + \sin^2 \theta)} = 0
\]

III. Let \( f(x,y) = x^2y + y^2 - 4y \).

1. (7 pts.) Find all of the critical point(s).

\[
f_x = 2xy = 0 \quad f_y = x^2 + 2y - 4 = 0
\]

\[
x = 0 \quad \text{or} \quad y = 0: \quad \text{When} \quad x = 0, \quad 2y - 4 = 0 \Rightarrow y = 2
\]

\[
\text{When} \quad y = 0, \quad x^2 - 4 = 0 \Rightarrow x = \pm 2
\]

Critical points are \((0,2), (-2,0), (2,0)\)

2. (7 pts.) Use the Second Partialss test to determine if each point you found in (1) is a local maximum, local minimum, or saddle point.

\[
f_{xx} = 2y \quad f_{yy} = 2 \quad f_{xy} = 2x \quad D = 4y - 4x^2
\]

For \((0,2), \quad D > 0 \quad f_{xx} > 0, \quad \text{so} \quad (0,2) \quad \text{is a local minimum}

For \((-2,0) \quad \text{and} \quad (2,0), \quad D < 0, \quad \text{so} \quad \text{these points are saddlepoints.}
3. (10 pts.) Suppose the function \( f(x, y) = x^2y + y^2 - 4y \) is bounded by the curves 
\( y = x^2, \ y = 3, \) and \( x = 0 \) in the first quadrant. Find the absolute maximum and absolute minimum of the function on this region.

The only critical point in the region is \((0, 2)\). List it.

From the picture below of the region we can list \((0,0), (0,3), \) and \((\sqrt{3},3)\).

![Graph](image)

Test boundaries: 
\[ x = 0 \Rightarrow f(y) = y^2 - 4y \Rightarrow f'(y) = 2y - 4 = 0 \Rightarrow y = 2. \]
This point is already on the list.

\[ y = 3 \Rightarrow f(x) = 3x^2 - 3 \Rightarrow f'(x) = 6x = 0 \Rightarrow x = 0. \]
\((0,3)\) is already on the list.

\[ y = x^2 \Rightarrow f(x) = 2x^4 - 4x^2 \Rightarrow f'(x) = 8x^3 - 8x = 0 \Rightarrow x = 0, 1, -1 \]
The only point in the region is \((1,1)\). Add it to the list.

Compute \( f(0,2), \ f(0,3), \ f(0,0), \ f(\sqrt{3},3), \ f(1,1) \). The biggest value is 6 at the point \((\sqrt{3},3)\)
The absolute maximum is 6. The smallest value is -4 at \((0,2)\). The absolute minimum is -4.

IV. Let \( f(x, y) = \ln(x^2 + y^4 + 1) \)

1. (5 pts.) What is the domain and range of \( f(x, y) \)?

   Domain\(\{(x, y) | x^2 + y^4 + 1 > 0\}\)\______________
   Range\(\{0, \infty\}\)\______________

2. (14 pts.) Find:

\[ f_x = \frac{2x}{x^2 + y^4 + 1} \]
\[ f_y = \frac{4y^3}{x^2 + y^4 + 1} \]
\[ f_{yx} = \frac{-8xy^3}{(x^2 + y^4 + 1)^2} \]
\[ f_{yy} = \frac{4y^3}{(x^2 + y^4 + 1)^2} \]
\[ f_{yy} = \frac{(x^2 + y^4 + 1)(12y^2) - 4y^3(4y^3)}{(x^2 + y^4 + 1)^2} \]
The period $T$ of a pendulum of length $L$ is $T = 2\pi \sqrt{\frac{L}{g}}$ where $g$ is the acceleration due to gravity. A pendulum is moved from the Canal Zone where $g = 32.09$ ft/sec, to Greenland, where $g = 32.23$ ft/sec. Because of the change in temperature the length of the pendulum changes from 2.5 ft. to 2.48 ft. Write an equation to approximate the change in the period of the pendulum. Do not evaluate.

$$dT = \pi \left( \frac{L}{g} \right)^{\frac{1}{2}} \frac{1}{g} \, dL + \pi \left( \frac{L}{g^2} \right)^{\frac{1}{2}} \left( -\frac{L}{g^2} \right) \, dg$$

$$dT = \pi \left( \frac{2.5}{32.09} \right)^{\frac{1}{2}} \frac{1}{32.09} (-0.02) + \pi \left( \frac{2.5}{32.09} \right)^{\frac{1}{2}} \left( -\frac{2.5}{(32.09)^2} \right) (.14)$$

**Extra Credit** (5 pts) Let $f$ be a function of $x$, $y$, and $z$ such that $f_x = \cos t$, $f_y = 2 \sin t$, $f_z = t$, $x = 2 \sin t$, $y = 1 - \cos t$, $z = t^2 - 5t$. Find all values of $t$ where $f$ has critical points.

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} = \cos t(2 \cos t) + 2 \sin t(\sin t) + t(2t - 5) = 0$$

$$2 \cos^2 t + 2 \sin^2 t - 5t = 2t^2 - 5t + 2 = 0 \Rightarrow (2t - 1)(t - 2) = 0$$

$$\Rightarrow t = \frac{1}{2}, 2$$