Problems on complex numbers

Note: Recall that $e^{i\theta} = \cos \theta + i \sin \theta$.

1. Express each of the following complex numbers in the form $a + bi$.

\[
\frac{1 - i}{2 + 2i}, \frac{1 + 2i}{2 - i}, \frac{8e^{i\pi/3}}{1 + i\sqrt{3}}, \left(\frac{\sqrt{3} + i}{2}\right)^9, \left(-4\sqrt{2} + 4i\sqrt{2}\right)^{1/3} \text{ (find all three roots)}.
\]

2. Find the roots of the following equations. Express your answers in the form $a + bi$.

(a) $2z^2 - 2z + 1 = 0$.
(b) $z^3 + 8i = 0$.

3. Given the equation

\[
\frac{1 + 2i}{x + iy + 1} = \frac{3}{x + 2iy},
\]

solve for the real values of $x$ and $y$.

4. Find the center and the radius of the sphere

\[
x^2 + y^2 + z^2 - 2x + 4y - 6z = 2.
\]