Math 2534    Homework sec 3.1-3.6

Instructions:  Either provide a proof or a counterexample for each of the following.

1) Suppose $a$ and $b$ are integers. The product $ab$ is odd if and only if, $a$ and $b$ are both odd.

2) The sum of any even integer and any odd integer is odd.

3) The product of any even integer and any other integer is even.

4) For all positive integers $n > 2$, if $n$ is prime then $n$ is odd.

5) The product of any two consecutive integers is even.

6) For all integers $a$, $b$, and $c$, if $a | b$ and $a | c$, then $a | (b + c)$.

7) For all integers $a$, $b$, and $c$, if $a | b$, then $a | bc$.
   (Prove by contrapositive and contradiction)

8) An integer $n$ is even if and only if $n^2$ is even.

9) If $x$ and $y$ are odd integers then $x + y$ is an even integer.
   a) Prove using the direct method ($p \rightarrow q$)
   b) Prove using the method of contrapositive ($\neg q \rightarrow \neg p$)
   c) Prove using the method of contradiction ($p \land \neg q$)

10) Given that $n$, $a$, $b$ are integers. If $n | a$ and $n | (a + b)$, Then $n | b$.

Use quantifiers to express each of the following sentences below symbolically. Negate each of the following sentences symbolically and then write the negation in a natural sounding sentence.

11) Not all integers are prime.

12) Every integer is even and a perfect square.