Prove the following:

1) Let $x$ and $y$ be Real Numbers.
   a) Use a direct proof to prove that if $x = y$, then $x^2 = y^2$.
   b) Redo the proof using proof by contradiction.
   c) Redo the proof using proof by contraposition.

2) Use the fifth postulate of Euclid and proof by contradiction to prove the following:
   Let $u$, $v$, and $w$ be distinct lines in a plane. If $u$ is parallel to $v$ and $v$ is parallel to $w$, then $u$ is parallel to $w$.

   **Fifth Postulate**: Given a line $m$ and a point $p$ not on $m$ (in a plane), there is exactly one line through $p$ and parallel to $m$.

3) Prove by contradiction: if $x$ is a real number and $x^3 + 4x = 0$, then $x = 0$.

4) Prove by contraposition that if $n^2$ is odd then $n$ is odd.

5) Use the method of contradiction to prove that $\sqrt{3}$ is irrational.

6) Prove that the product of three consecutive integers is divisible by three using the Quotient Remainder Theorem.

7) Prove by contraposition that for all integers $a$, $b$, and $c$, if $a$ does not divide $bc$, then $a$ does not divide $b$.

8) Prove by contraposition that for all integers $m$ and $n$, if $m + n$ is even then $m$ and $n$ are both odd or both even.