Quiz answers:

Use PMI to prove the following:

If \( a_1 = 1, a_2 = 1, a_n = 2a_{n-1} + 3a_{n-2}, n \geq 3 \) then \( a_n < 3^n \) for all natural numbers

Proof: I will validate the hypothesis for

n = 3.

\[ a_3 = 2a_2 + 3a_1 = 2(1) + 3(1) = 5 \]

\( 5 < 3^3, 5 < 27 \)

n = 4

\[ a_4 = 2a_3 + 3a_2 = 2(5) + 3(1) = 13 \]

\( 13 < 3^4, 13 < 81 \)

I will now assume the hypothesis is valid up to some arbitrary n and prove true for n+1

i.e. I will show that \( n+1 < 3^{n+1} \)

Proof: Consider the n + 1 term:

\[ a_{n+1} = 2a_n + 3a_{n-1} \]

since we have assumed n to be true then n – 1 must also be in the truth set and we have that

\[ a_{n+1} = 2a_n + 3a_{n-1} < 2(3^n) + 3(3^{n-1}) = 2(3^n) + 3^n = 3^n = 3^{n+1} \]

I have shown that \( a_{n+1} < 3^{n+1} \). Therefore since I assumed true for some arbitrary n and proved true for n+1, the hypothesis is true for all natural numbers \( n > 2 \).