Worksheet for Math 2534 Chapter 7 – 10

1) Are the following functions one to one and onto? Is the inverse function well defined? In not then make adjustments necessary for the inverse to be valid.
   a) \( F(x) = 10x + 4 \) defined on \( \mathbb{R} \)
   b) \( F(x) = \frac{6x - 1}{x + 3} \) defined on \( \mathbb{R} - \{-3\} \)
   c) \( F(x) = \ln(x - 4) \) Defined on \( \mathbb{R} - \{4\} \)

2) Prove the following: Given \( A, B \) are sets and \( f \) maps \( A \) to \( B \) and \( g \) maps \( B \) to \( A \),
   A) If the functions \( f \) and \( g \) are both “one to one” then the composition \( (g \circ f) \) is also one to one.
   b) If the functions \( f \) and \( g \) are both “onto” then the composition \( (g \circ f) \) is also onto.

3) Are the following Equivalence Relations on \( A = \{1,2,3,4,5\} \)?
   \( R_1 = \{(1,1),(2,2),(3,3),(3,4),(4,3),(4,4)(5,5)\} \)
   \( R_2 = \{(1,2),(1,4),(1,5),(2,4),(2,5),(3,4),(3,5),(4,5)\} \)
   \( R_3 = \{(1,3),(2,4),(1,5),(3,1),(3,5),(4,2),(5,1),(5,3)\} \)
   \( R_4 = \{(1,1),(1,3),(1,5),(2,2),(3,1),(2,4),(3,3),(3,5),(5,3),(5,1),(4,2),(4,4)(5,5)\} \)

   Draw a directed graph for each relation given above.

4) Is the function \( f : \mathbb{Z} \mod 4 \to \mathbb{Z} \mod 4 \) given by \( f[x] = [7x] \) a bijection? Is the inverse function well defined?

5) Verify that the mapping from the positive integers into the positive integers given by \( f(z) = z \mod 4 \) is an equivalence relation. Restated, this is the same as saying \( 4|z - y \) where \( y = f(z) \). [Notice that this equivalence relation will partition the set of integers into equivalence classes. Describe the equivalence classes]

6) Prove that the relation \( R \) given by \( (a,b) \in R \iff ab = bc \) is an equivalence relation when \( a, b \) are integers. Describe the equivalence classes that are created.

7) If an Equivalence relation is defined by the following pair wise disjoint sets. Then express this relation as order pairs.
   \[ R = \{1, 2\} \cup \{3, 5\} \cup \{4\} \]

8) If \( f \) maps finite sets \( A \) to \( B \) and \( n(A) > n(B) \) prove that \( f \) is one to one iff \( f \) is onto.

9) If there are 500 students in one lecture hall. How many are guaranteed to have the same birthday. (Explain using the Pigeonhole Principle.)
10) Let \( A = \{2, 4, 6\} \). Define a binary relation from \( A \) to \( A \) as follows:
\[ R = \{(2, 6), (4, 2), (6, 4)\} \]

a. Is 4 R 6?
b. Is R a function?
c. Does R have an inverse? If so what is it?
d. Draw the directed graph of R.
e. Is R reflexive? Explain.
g. Is R transitive? Explain.
h. Is R antisymmetric? Explain.
i. Is R an equivalence relation? Why or why not?
j. Is R a partial order relation? Why or why not?

11). D is the binary relation defined on \( R \) as follows:
\[ \forall (x, y) \in D, \ x \text{Dy} \leftrightarrow xy > 0 \]

a. Is -3 R 4?
b. Is D a function?
d. Is D symmetric? Explain.
e. Is D transitive? Explain.
g. Is D antisymmetric? Explain.
h. Is D an equivalence relation? Why or why not?
i. Is D a partial order relation? Why or why not?

12) \( R \) is the binary relation defined on \( A = \{1, 2, 3, 4, 6, 12\} \) as follows:
\[ \forall x, y \in A, \ x \text{Ry} \leftrightarrow x \text{ exactly divides } y \]
a. Is R a function?
b. Does R have an inverse? If so what is it?
e. Is R transitive? Explain.
g. Is R antisymmetric? Explain.
h. Is R an equivalence relation? Why or why not?
i. Is R a partial order relation? Why or why not?

13) Given that R is a relation and A, B, C are sets such that \( A \text{RB} \) iff \( A \cap C = B \cap C \), Verify that R is an equivalence relation.