**Answer to Multivariable worksheet #12**

1) a) \( y = \frac{C}{x^2} \) substitute in the given values for C and graph each equation. Refer to worksheet #11 for other examples.
   b) \( f(x, -4) = -4x^2 \)

2)  
   \[
   R(x, y) = -x^2 - 3y^2 + 16x + 42y
   \]
   
   \[
   R_x = \frac{\partial R}{\partial x} = -2x + 16 = 0 \quad x = 8
   \]
   
   \[
   R_y = \frac{\partial R}{\partial y} = -6y + 42 = 0 \quad y = 7
   \]
   
   \[
   R_{xx} = -2, \quad R_{yy} = -6
   \]

   The optimal point is at \((8, 7, R(8, 7))\)

   Second partials test
   \( D = (-2)(-6) - (0)^2 > 0 \) and \( Rxx < 0 \), Therefore we do have a relative max at \( x = 8 \) and \( y = 7 \)

3)  
   \[
   P_x = 2x - y - 15 = 0
   \]
   
   \[
   P_y = 2y - x - 30 = 0
   \]
   
   \[
   P_{xx} = 2, P_{yy} = 2
   \]

   Now set up a system of equations and solve for \( x \) and \( y \).
   
   \[
   \begin{cases}
   -x + 2y = 30 \\
   2x - y = 15
   \end{cases}
   \]

   \[
   \begin{bmatrix}
   -1 & 2 & 30 \\
   2 & -1 & 15
   \end{bmatrix} \rightarrow work \rightarrow \begin{bmatrix}
   1 & 0 & 20 \\
   0 & 1 & 25
   \end{bmatrix}
   \]

   so the possible optimal point is \((20, 25, P(20, 25))\)

   We need to do the 2\(^{nd}\) Partials test to determine if it is a max, min, or saddle point

   \[
   D = P_{xx} P_{yy} - (P_{xy})^2 = (2)(2) - (-1)^2 > 0
   \]

   \[
   and \quad P_{xx} > 0
   \]

   so the function \( f(x, y) \) have a relative minimum at this point.
4) 
\[ f_x = 3x^2 - 27 = 0, \quad x = \pm 3 \]
\[ f_y = 2y - 20 = 0, \quad y = 10 \]
so the possible optimal points occur at (3, 10) and (-3, 10)
\[ f_{xx} = 6x, \quad x = 3 \rightarrow f_{xx} = 18 > 0 \]
\[ f_{xx} = 6x, \quad x = -3 \rightarrow f_{xx} = -18 < 0 \]
\[ f_{yy} = 2 \]
Now use the second partials test to determine if the points are max, min, or saddle pt.
For the point (3, 10) we have that \[ D = (18)(2) - (0)^2 > 0 \] and \[ f_{xx} = 18 > 0 \] so this point will give a relative maximum.
For the point (-3, 10) we have that \[ D = (-18)(2) - (0)^2 < 0 \] so this point is a saddle point.

5) 
\[ a) \quad C_x \bigg|_{(20,15)} = 6x + y = 6(20) + 15 = 135 \quad \text{Meaning is: If the number of robot dogs is increased from 20 to 21 and robot cats are held steady at 15, then total cost will increase by $135.} \]
\[ b) \quad C_y \bigg|_{(10,18)} = 2y + x = 2(18) + 10 = 46 \quad \text{Meaning is: If the number of robot cats is increased from 18 to 19 and robot dogs are held steady at 10, then total cost will increase by $46} \]
\[ c) \quad C(8, 5) = $407. \quad \text{To test to see if the point is a critical point you must evaluate the first partial derivative to see if you get zero.} \]
\[ C_x \bigg|_{(8,5)} = 6x + y = 6(8) + 5 \neq 0 \quad \text{and} \quad C_y \bigg|_{(8,5)} = 2y + x = 2(5) + 8 \neq 0 \]
Since the first derivatives are not zero, the point (8, 5) cannot be a critical point.

6) 
\[ \frac{\partial Q_A}{\partial P_B} = -3, \quad \frac{\partial Q_B}{\partial P_A} = -2 \quad \text{Since they are negative the products are complementary.} \]
(See module for explanations)