Answers to Test 4A:

1) \[ y = 4x^2 + \frac{z}{2}, \quad y = 4x^2 + 10, \quad y = 4x^2 + 8, \quad y = 4x^2 + 4 \]

2) \[
P(x, y) = 72x - 32x^2 - y^2 + 14y + 4000
\]
\[
P_x = 72 - 6x = 0, \quad x = 12
\]
\[
P_y = -2y - 14 = 0, \quad y = 7
\]
\[
P_{xx} = -6, \quad P_{yy} = -2
\]
\[
D = P_{xx}P_{yy} - (P_{xy})^2 = (-6)(-2) - (0)^2 > 0
\]
Since \( D > 0 \) and \( P_{xx} < 0 \), we can conclude that the point \( x = 12 \) and \( y = 7 \) will produce maximum profit.
If the company sells 12 hundred Hokie masks and 7 hundred wigs they will make a maximum profit of \( P(12, 7) \) in dollars.

3)
   a) \( P(2,4) = -4 \). If the Market Square sells 2 pots of H.O. and 4 pots of M.M., they will still be in the red by $4.
   
   b) \( P_{y|_{x=1}} = 4x - 8 = 4(1) - 8 = -4 \) If the number of pots of MM sold is increased from 3 to 4 pots and the H.O. is held constant at 1 pot, total profit will decrease by $4.
   
   c) \( P(x, 5) = -x^2 + 20x - 40 \)

4) Using the second partials test, Megan decides that the point is really just a saddle point.
\[
D = (2)(7) - (-5)^2 = 14 - 25 < 0
\]

5) a) the point located at August 2000 is a local minimum.
   
   b) On the table of values draw a line connection all the values of \( C = 50 \) from left to right.
6) a) 

\[ L(x, y) = 2x^2 - 2xy - 12y + 20x + 12000 - \lambda (x + y - 200) \]

\[ L_x = 4x - 2y + 20 - \lambda = 0 \]

\[ L_y = -2x - 12 - \lambda = 0 \]

\[ L_\lambda = -x - y + 200 = 0 \]

Solve for \( \lambda \) in \( L_x \) and \( L_y \) and set them equal to each other to get the following:

\[ 4x - 2y + 20 = -2x - 12 \]

\[ y = 3x + 16 \]

Now substitute into \( L_\lambda \) to get 

\[ -x - (3x + 16) + 200 = 0 \]

and solve for \( x \) to get \( x = 46 \) and \( y = 154 \)

b) Solve for \( \lambda = -2x - 12 = -2(46) - 12 = -104 \)

c) If the constraint is increased from 200 to 201, then the minimum value $2432 will decrease to $2328.

7) 

\[ f_{xx} = -3(4) + 2(-1) = -14 \]

\[ f_{yy} = 4(4) + 1 = 17 \]

Since the second derivative with respect x is negative, that indicates that the graph is probably concave down. The second derivative with respect to y is positive, which indicates that the graph is probably concave up at this point. This point is probably a saddle point.