Answers to Test 2B

1) (16pts)
\[ \frac{dw}{dt} = kw \]
\[ \int \frac{1}{w} dw = \int k dt \]
\[ \ln w = kt + C \]
\[ e^{\ln w} = e^{kt+c} = e^{kt}e^c \]
\[ w = Be^{kt} \]
\[ w = 40e^{(\ln 20)/2} \]

2) (6pts) \( \frac{dP}{dt} \) is the rate of change in population per year.
There is a 3% change in population each year
200 is the limit to the number of geese that can live at the pond at one time
P is the present population of geese at the pond.

3) (12pts) \( y = \ln[(1/2)x^2 - 1] \) and \( x > 1 \)

4) (8pts) \( FV = e^{12(8)} \int_0^8 1200e^{-12t} dt \)

5) (12pts) \( PV = \int_0^{10} 70000e^{-0.09t} dt = \frac{70000}{-0.09} \left[ \frac{1}{e^{0.09(10)}} - 1 \right] \)

6) (10pts) Bears sold =
\[ \int_0^{\infty} 360e^{-0.04t} dt = \text{Limit}_{r \to \infty} \int_0^{\infty} 360e^{-0.04t} dt = \text{Limit}_{r \to \infty} \frac{360}{-0.04} e^{-0.04t} \bigg|_0^r \]
\[ = \text{Limit}_{r \to \infty} \frac{360}{-0.04} \left[ \frac{1}{e^{0.04r}} - \frac{1}{e^0} \right] = \frac{360}{-0.04} [0 - 1] = 9000 \]

Therefore 9000 bears will be sold and there will be 300 bears left over.

7) (20pts)
   a) \( p = 13 \) so the price is \$13000
   b) \( CS = \int_0^1 q^2 - 8q + 20 - 13dq = 3.3 \) thousand dollars
   c) the point associated with Social Gain is the equilibrium point where
   \( q = 2 \) and \( p = 8 \). Therefore the point of equilibrium occurs when
   2 million items are made and sold at $8000 per unit.
   \( SG = \int_0^2 (q^2 - 8q + 20) - (q^2 + 2q) dq = 20 \) thousand dollars