Answers to PMI 1 worksheet

Warning: 😊 I am not doing answers or complete write ups for all the problems on this sheet. I will do a couple of complete write ups, but I expect to see complete write ups on anything you hand in to be graded.

The correct write up will involve complete sentences and the following outline:
1) Statement of Hypothesis
2) Verification of base cases (Elements in the truth set)
3) Assume true for n
4) Prove true for n+1 (you must show the reader what the expected end results will be)
5) Body of Proof
6) Statement of conclusion that will usually go something like this: Since I have assumed true for arbitrary n and proved true for n+1, the Hypothesis is true for all natural numbers for n > ??.

3) I will prove using the Principle of Math Induction that \(2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2\) for all natural numbers.
I will first verify that the hypothesis is true for at least one value of n.
Consider the value n=1.
\[
\begin{align*}
2^1 &= 2^{1+1} - 2 \\
2 &= 4 - 2 \\
2 &= 2
\end{align*}
\]
Now I will assume true for some arbitrary n and prove true for n+1. I will show that \(2 + 2^2 + 2^3 + \ldots + 2^{n+1} = 2^{n+2} - 2\)

Proof: Since we assume that the hypothesis is true for n, we have that
\[
2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2
\]
Now add the n + 1 term to both sides of the equation to get
\[
2 + 2^2 + 2^3 + \ldots + 2^n + 2^{n+1} = 2^{n+1} - 2 + 2^{n+1}
\]
\[
= 2 * 2^{n+1} - 2
\]
\[
= 2^{n+2} - 2
\]
Which gives me the results I needed as shown above. Since I have assumed true for some arbitrary natural number n and prove true for n+1, the hypothesis is true for all natural numbers.
5) I will prove using PMI that $1 + 2^n < 3^n$ for all natural numbers $n \geq 2$.
   I will show that the hypothesis is true for at least one value of $n$.
   Consider $n = 2$: $1 + 2^2 < 3^2$ to get $5 < 9$.

   Now I will assume true for some arbitrary $n$ and prove true for $n+1$. I will show that $1 + 2^{n+1} < 3^{n+1}$

   Proof: Consider $1 + 2^{n+1} = 1 + 2*2^n < 1 + 2*3^n < 1 + 3*3^n = 1 + 3^{n+1}$
   Conclusion: Since I have assumed true for some arbitrary $n$ and proved true for $n + 1$, the hypothesis is true for all natural numbers $n \geq 2$.

8) Incomplete write to show body of proof only.
   1) Give statement and method of proof ( $3^n < n!$ for $n > 4$)
   2) Verify for at least one value of $n > 4$
   3) State clearly assumptions, givens, and show what you are trying to prove
   4) Body of Proof:
      Consider $3^{n+1} = 3*3^n < 3*n! < n*n!$ (since $n > 4$) $< (n + 1)*n! = (n + 1)!$
   5) Conclusion

9) Incomplete write to show body of proof only.
   1) Give statement and method of proof ( $2^n \geq n^2 + n$ for $n > 4$)
   2) Verify for at least one value of $n > 4$
   3) State clearly assumptions, givens, and show what you are trying to prove
   4) Body of Proof:
      Consider $2^{n+1} = 2*2^n \geq 2*(n^2 + n) \geq 2n(n + 1) = n(n + 1) + n(n + 1) \geq n(n + 1) + 2(n + 1)$ (since $n > 4$) $= (n+1)(n+2) = (n+1)((n+1)+1)$
      $= (n+1)^2 + (n+1)$
   5) Conclusion

11) Incomplete write to show body of proof only.
   1) Give statement and method of proof ( $5\left(2^{2n-1} + 3^{2n-1}\right)$)
   2) Verify for at least one value of $n$
   3) State clearly assumptions, givens, and show what you are trying to prove
   4) Body of Proof:
      $2^{2n+1} + 3^{2n+1} = 2^2 * 2^{2n-1} + 3^2 * 3^{2n-1} = 2^2 * 2^{2n-1} + (2^2 * 3^{2n-1} - 2^2 * 3^{2n-1}) + 3^2 * 3^{2n-1} + 3^2 * 3^{2n-1}$
      $= 2^2(2^{2n-1} + 3^{2n-1}) + 3^{2n-1}(3^2 - 2^2)$
      Notice that 5 divides the first term by assumption for $n$ and it divides the second term since $9-4 = 5$.
   Conclusion is :........