Answers to Math 2534 Test 3:

Problem 1:
A) To prove that \( f(x) \) is one to one we will assume \( f(a) = f(b) \) and show that \( a = b \).

Consider
\[
\frac{8a}{a-3} = \frac{8b}{b-3}
\]
\[
8a(b-3) = 8b(a-3)
\]
\[
8ab - 24a = 8ab - 24b
\]
\[-24a = -24b
\]
\[a = b
\]

B) To prove that \( f(x) \) is onto we will show that for each \( b \) in the co-domain there exist an \( a \) in the domain so that \( f(a) = b \).

\[
b = \frac{8a}{a-3}
\]
\[
b(a-3) = 8a
\]
\[
ba - 3b = 8a
\]
\[
ba - 8a = 3b
\]
\[a = \frac{3b}{b-8}
\]

\( f(x) \) is not onto since no element in the domain maps to \( b = 8 \).

C) In order to create an inverse function the \( f: \mathbb{R} - \{3\} \) into \( \mathbb{R} - \{8\} \) and
\[
f^{-1}(x) = \frac{3x}{x-8}
\]

Problem 2: a) \( f(S) \) is a function since it is defined for all elements in the domain and for each input there is only one output (it is also one to one)

b) \( G(f \circ g) = \{(a, c), (b, b), (c, d), (d, a)\} \)
c) \( G(f \circ g)^{-1} = \{(c, a), (b, b), (d, c), (a, d)\} \)

Problem 3: Theorem: If \( f(x) \) is invertable then \( f(x) \) is one to one.

Proof: To show that \( f(x) \) is one to one we will show that if \( f(a_1) = f(a_2) \), then \( a_1 = a_2 \). Consider the following
\[
a_1 = f^{-1}(b_1) = f^{-1}(f(a_1)) = f^{-1}(f(a_2)) = (f^{-1} \circ f)(a_2) = a_2
\]
where \( f(a_1) = b_1 \) since the composition of \( f \) and its inverse is the identity function. Therefore since we have shown that \( a_1 = a_2 \), we have shown that \( f(x) \) is one to one.
Problem 4: To show that R is an equivalence relation, we must show that it is reflexive, symmetric, and transitive.

Reflexive: Does aRa? Does $7|a-a$? If $7q = a-a$ then $7q = 0$ and $q = 0$.

Symmetric. If aRb then show that bRa. Since $7q = b-a$, then we have that

$7k = a-b$ where $k = -q$ and $k$ is an integer. Therefore, by definition of division we have that $7$ divides $a-b$ and bRa.

Transitive: If $aRb$ and $bRc$, we will show that $aRc$.

We have that $7q = b - a$ and $7d = c - b$, now add them together to get $7q + 7d = (b - a) + (c - b) = (c - a)$ so we have that $7k = (c - a)$ where $k = q + d$ is an integer and by definition of division $aRc$.

Therefore R is an equivalence relation.


Problem 5: In order to show that R is transitive I need to show that if ARB and BRC then ARC.

Since ARB, there is a bijection f that maps from A to B and since BRC there is a bijection g that maps from B to C. We have proved earlier that if f and g are both bijections then the composition of g and f, $g \circ f$ is also a bijection and maps A to C so ARC. Therefore R is transitive.

Problem 6: a) $R = \{(3,3), (4,4), (6,6), (12, 12), (16, 16), (24,24), (3, 6), (3,12), (3, 24), (4,16), (4, 12), (4, 24), (6, 12), (6,24)\}$

b) Diagram in class

Problem 7: Anti symmetric: ARB and BRA if and only if $A=B$.

If aRb then $a = b^k$ and if bRa then $b = a^n$, where $k$ and $n$ are positive integers. Since $a = b^k$, then $a = (a^n)^k = a^{nk}$ and the only way this can be true is for $n=1, k=1$ and $a = b$. So R is anti-symmetric.

Problem 8: Domain and Range is A

Problem 9: Suppose the first person choose a card and it is black, then a second person chooses and card. This card could be black and you are done or the card could be red. Then a third person chooses a card. It must be black or red. Either way, you have two cards of the same color. Therefore 3 people must be at the party to guarantee that at least two people will each choose a card of the same color.