Math 2534 Homework 2 Solutions

Problem 1: Use Algebra of Logic to prove the following:

\[(p \rightarrow q) \land \sim p \rightarrow \sim (p \rightarrow q) \equiv p\]

Proof:

\[\[(p \rightarrow q) \land \sim p \rightarrow \sim (p \rightarrow q) \equiv \quad \text{given}\]
\[\sim [(\sim p \lor q) \land \sim p] \lor \sim (\sim p \lor q)] \equiv \quad \text{Implication Law}\]
\[\sim (\sim p \lor q) \lor (\sim p \lor \sim (p \land q)) \equiv \quad \text{DeMorgan’s Law} \]
\[\sim (p \land \sim q) \lor p \lor (p \land \sim q) \equiv \quad \text{DeMorgan's Law and Double negative} \]
\[p \lor [(p \land \sim q)] \equiv \quad \text{Absorption Law} \]
\[p \quad \text{Absorption Law}\]

Therefore

\[(p \rightarrow q) \land \sim p \rightarrow \sim (p \rightarrow q) \equiv p\]

Problem 2: Using algebra of logic put the following in Disjunctive Normal Form:

\[(p \rightarrow q) \rightarrow [(p \land r) \rightarrow (q \land r)] \equiv \quad \text{Given}\]
\[\sim (\sim p \lor q) \lor [(\sim (p \lor r) \lor (q \land r))] \equiv \quad \text{Implication Law}\]
\[(p \land \sim q) \lor [(\sim p \lor \sim r) \lor (q \land r)] \equiv \quad \text{Demorgan’s Law & Double Neg Law} \]
\[(p \land \sim q) \lor (\sim p) \lor [(\sim r \lor (q \land r))] \equiv \quad \text{Associative Law} \]
\[(p \land \sim q) \lor (\sim p) \lor [(\sim r \land q) \lor (\sim r \lor r)] \equiv \quad \text{Distributive Law} \]
\[(p \land \sim q) \lor (\sim p) \lor [(\sim r \lor q) \lor T] \equiv \quad \text{Negation Law} \]
\[(p \land \sim q) \lor (\sim p) \lor (\sim r \lor q) \equiv \quad \text{Identity Law for } \land \]
\[(p \land \sim q) \lor (\sim p \land \sim p) \lor (\sim r \land \sim r) \lor (q \land q) \equiv \quad \text{Idempotent Law}\]

Problem 3: Put the following into symbolic implication form. Define all your variables.

a) I will clean up only if you help.

Let I be the statement “I will clean up” and Y be the statement “you help”

\[I \rightarrow Y\]

b) Game will be postponed since it is raining.

Let G be the statement “Game is played” and R be the statement “It is raining”

\[R \rightarrow \sim G\]

c) I will not go to the movie or I will not study.

Let M be the statement “I will go to the movie” and S be the statement “I will study”.

\[\sim M \lor \sim S \equiv M \longrightarrow S\]
Problem 4: If Anna goes to Roswell, New Mexico, she might see an alien space ship.  
Let R be the statement “Anna goes to Roswell” and Let S be the statement “Anna might see an alien space ship.  \( R \rightarrow S \)

1) Rewrite the above sentence in inverse form. \( \sim R \rightarrow \sim S \)
   If Anna does not go to Roswell, there is no possibility she will see an alien space ship.

2) Rewrite the above sentence in converse form. \( S \rightarrow R \)
   If there is a possibility that Anna might see an alien space ship, then she went to Roswell.

3) Rewrite the above sentence in contrapositive form. \( \sim S \rightarrow \sim R \)
   If there is no possibility that Anna might see an alien space ship then she did not go to Roswell.

Problem 5: Determine if the following arguments are valid and justify your conclusion.  
Put each argument into symbolic logic and define all variables. In justifying your conclusion be sure to indicate what is the sufficient condition and what is the necessary condition.

a) If you are in the Marching Virginians, then you must go to the game.  
   You went to the game.  
   Therefore you are in the Marching Virginians.
   Solution: Let M be the statement “you are in the Marching Virginians”.  
   Let G be the statement “you go to the game”.  
   \( M \rightarrow G \)
   \( G \)
   \( \therefore M \)
   This is not a valid argument since the necessary condition does not guarantee the sufficient condition. This is the Converse Error.

b) If the test is Thursday, you will miss the game.  
   You did not miss the game.  
   Therefore you did not have a test.
   Solution: Let T be the statement “the test is Thursday”.  
   Let G be the statement “you miss the game”.  
   \( T \rightarrow G \)
   \( \sim G \)
   \( \therefore \sim T \)
   This is a valid argument since this is the contrapositive and in equivalent to the original implication.
Problem 6: P, Q and R represent the following statements:

- P: Jim is a CS Major
- Q: Anne is an EE Major
- R: Laura is an Environmental Science Major
- M: Charlie is a Math Major

Assume that the expression \((P \lor \sim R) \rightarrow (Q \land M)\) is false and that R is true and M is true.

Put the following statements into implication form and determine if the sufficient and necessary conditions are true or false and if the implication is true or false.

a) Anne is a EE Major or Charlie is not a Math Major.
b) Jim is a CS Major and Anne is not an EE Major.
c) Only if Anne is a EE Major is Jim a CS Major

SOLUTION: We are given that the implication is false and this can only happen if the sufficient condition \((P \lor \sim R)\) is true and the necessary condition \((Q \land M)\) is false. We also know that M is true which means that Q is false since it is part of a conjunction. We know that R is true so \(\sim R\) must be false. Since it is part of the disjunction that makes up the sufficient condition with a true value, we know that P is true.

\[
\begin{align*}
a) & \quad Q \lor \sim M \equiv \sim Q \rightarrow \sim M \equiv F \rightarrow \sim T \equiv T \rightarrow F \equiv F \\
b) & \quad P \land \sim Q \equiv \sim P \rightarrow \sim Q \equiv \sim T \rightarrow \sim F \equiv F \rightarrow T \equiv T \\
c) & \quad P \rightarrow Q \equiv T \rightarrow F \equiv F
\end{align*}
\]

Problem 7: Put the following into symbolic logic using the appropriate quantifier and give your domain and predicate.

1) Any CS major loves Discrete Math. Let C be the set of all CS majors where \(x\) is an element in C and \(P(x) = x\) loves discrete math.

\[\forall x, x \in C \rightarrow P(x)\]

2) Hardly anyone likes bugs. Let P be set of all people where \(x\) is an element in P and \(B(x) = x\) likes bugs.

\[\exists x | x \in P \land B(x)\]
**Problem 8:** Below is logic puzzle by Lewis Carroll. In these puzzles he strings together a list of implications and the job of the reader is to use all the listed implications to arrive at an inescapable conclusion. Put all statements into symbolic implication form. Determine the conclusion and justify your reasoning using sentences.

1) Promise breakers are untrustworthy
2) Wine drinkers are very communicative.
3) A man who keeps his promises is honest.
4) No teetotalers are pawnbrokers.
5) One can always trust a very communicative person.

Use the following Variables for you symbolic argument so that we all have the same notation.

- **P:** Keeps Promises
- **T:** Is Trustworthy
- **W:** Drinks Wine
- **C:** Very communicative
- **H:** Is honest
- **A:** Is a pawnbroker

**Solutions:**

1) \( \neg P \rightarrow \neg T \)
2) \( W \rightarrow C \)
3) \( P \rightarrow H \)
4) \( \neg W \rightarrow \neg A \)
5) \( C \rightarrow T \)

Using the contrapositive we have that \( \neg P \rightarrow \neg T \equiv T \rightarrow P \) and \( \neg W \rightarrow \neg A \equiv A \rightarrow W \)

Using the argument form of transitive we have that \( A \rightarrow W \rightarrow C \rightarrow T \rightarrow P \rightarrow H \)

Therefore Pawnbrokers are honest.