Math 2534  Solution  Homework 1  Fall 2013

Problem 1:
Use Truth Tables to verify each of the following:

a) \( \sim (\sim p \land q) \equiv p \lor \sim q \)

\[
\begin{array}{cccc}
p & q & \sim p & \sim q & \sim (\sim p \land q) & p \lor \sim q & \sim (\sim p \land q) \equiv p \lor \sim q \\
T & T & F & F & F & T & T \\
T & F & F & T & F & T & T \\
F & T & T & F & T & F & T \\
F & F & T & T & F & T & T \\
\end{array}
\]

Notice that columns 6 and 7 have identical truth values and therefore \( \sim (\sim p \land q) \equiv p \lor \sim q \).

b) \( \sim [(\sim q) \lor (\sim p)] \equiv p \)

\[
\begin{array}{cccc}
p & q & \sim p & \sim q & \sim q & \sim p & \sim [(\sim q) \lor (\sim p)] & \sim [(\sim q) \lor (\sim p)] \equiv p \\
T & T & F & T & F & T & T & T \\
T & F & F & F & F & T & T & T \\
F & T & F & T & T & F & T & T \\
F & F & T & T & T & F & T & T \\
\end{array}
\]

Notice that columns 1 and 6 have identical truth values and therefore \( \sim [(\sim q) \lor (\sim p)] \equiv p \).

Problem 2:
Given the following statements:

P: Grant is a freshman
Q: Ryan is not a sophomore
R: Charles is graduating this semester.

The statement P is true and the statement Q and R are false. Express each of the following compound sentences in symbolic logic and then use truth values to determine the overall truth value of each compound statement.

1) Neither Grant is a freshman nor Ryan is sophomore.
\( \sim p \land q \equiv \sim T \land F \equiv F \land F \equiv F \)

2) Either Charles is not graduating or Grant is not a freshman.
\( \sim R \land \sim P \equiv \sim T \land F \equiv T \land F \equiv T \)

3) Charles is graduating and Ryan is a sophomore but Grant is not a senior.

Let M be the statement that "Grant is not a Senior" which is True
\( R \land \sim Q \land M \equiv F \land T \land T \equiv F \)
**Problem 3:**

Use the algebra of logic to simplify and verify the following:

Theorem: \( \sim [\sim (p \lor q) \lor \sim q] \lor \sim q \equiv T \)

Proof:

\[
\begin{align*}
\sim [\sim (p \lor q) \lor \sim q] \lor \sim q & \equiv \quad \text{Given} \\
[\sim (p \lor q) \land \sim q] \lor \sim q & \equiv \quad \text{DeMorgan's Law} \\
[(p \lor q) \land q] \lor \sim q & \equiv \quad \text{Double Negative} \\
q \lor \sim q & \equiv \quad \text{Absorption Law} \\
T & \equiv \quad \text{Inverse Law}
\end{align*}
\]

Therefore \( \sim [\sim (p \lor q) \lor \sim q] \lor \sim q \equiv T \)

**Problem 4:**

The **nand** operator denoted by ‘\( \& \)’ is defined by \( \text{PIQ} \Leftrightarrow \sim (P \land Q) \).

1) Set up the truth table for \( \text{PIQ} \)

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
p & q & p \land q & \sim (p \land q) & p | q \\
T & T & T & T & F \\
T & F & F & T & T \\
F & T & F & T & T \\
F & F & F & T & T \\
\end{array}
\]

2) Express \( \sim (p \land p) \) in terms of “nand” only.

Notice that by the idempotent Law we have that \( \sim (p \land p) \equiv \sim p \).

We also have by definition that \( \sim (p \land p) \equiv p | p \). Therefore we also have that \( \sim p \equiv p | p \)

**Problem 5:**

Determine which of the following statements is the Exclusive Or and which is the Inclusive Or.

1) Either you go skiing or you play paint ball this afternoon. **Exclusive OR**

2) To be eligible for international study you must have a 3.5 grade average or 3 letters of recommendation from professors in your major. **Inclusive OR**

3) You will go to graduate school at VA Tech or UNC next semester. **Exclusive OR**

4) You will clean your apartment or go to the grocery store this weekend. **Inclusive OR**
Problem 6:
logic puzzle.
Solve the following puzzle and then write up a paragraph explaining your reasoning. Use your own judgment on how to write this up. It needs to be clear to anyone who tries to follow your reasoning. You may use a table to help you with your reasoning but do not include it in your write up.

A Day at the Beach

Four cousins spent one summer day at Edisto Beach in South Carolina. Two were two years old. Of the other two, one was five years old and the other was four years old. Determine the name and age of each child and the beach activity of each. One was looking shells, one was building a sand castle and two were playing in the sea. All statements are true.

1) Neither Oliver nor the one who was building the sand castle is the oldest.
2) Neither of the two youngest was building the sandcastle.
3) Grant is older than the boy with whom he is playing in the sea.
4) Neither Ryan nor the boy who is four were looking for shells.
5) Jack, who is not the oldest, was not playing in the sea; nor was Oliver who is not older than Jack.

Solution:

By statements 1) and 5) we know that Oliver was not playing in the sea or building sand castles, so he was hunting shells. By statement 3) we know that Grant played in the sea. By statement 5) we know that Jack, who is not the oldest, is older that Oliver. We also know that Jack is not playing in the sea nor is he hunting sea shells. Therefore Jack is building a sand castle which means that Ryan is playing in the sea. By statement 3) we know that Grant is older than Ryan and that by statement 4) Ryan is not 4. Therefore Grant must be the oldest and is 5. Since Ryan is not 4 and Oliver is younger than Jack, we have that Jack is 4. Therefore Ryan and Oliver are 2.