Math 2534 Test 1A  Spring 2009  Name

Do not put answers on this test paper. Show all work and justify your answers in complete sentences when appropriate. NO ELECTRONIC DEVICES of any kind.

Problem 1:
Put the following statement into symbolic logic with multiple quantifiers.
Each student at VT takes at least one math course.

Problem 2
If the operation of "nor" is defined to be \( p \downarrow q \equiv \neg (p \lor q) \). Then use the algebra of logic to prove the following theorem and justify each step.
Theorem: \( (p \downarrow q) \downarrow (p \downarrow q) \equiv p \lor q \).

Problem 3
Are the following two statements equivalent? Justify your answer using symbolic logic along with some English sentences.

a) I will not dance or I will not sing.
b) I will sing only if I do not dance.

Problem 4: Using one or more of the following previously proved theorems below, prove the following Theorem:
If \( a > 2 \) is a prime number and \( (b+1) \) is an even number, then \( a^4 + b \) is even.

List of previously proved theorems:
Refer to needed theorems only. Do not try to reprove any of these theorems.

- Two consecutive integers have opposite parity
- The product of two even (odd) integers is even (odd).
- The sum of any two odd integers or any two even integers is even.
- Any prime number greater than 2 is an odd integer.
- The sum of any two integers is an integer.
- The sum of an even and odd integer is odd.

Problem 5: Are the following arguments valid? Justify your conclusion.

1) An apple a day keeps the doctor away. Since Fred is sick, he did not eat an apple everyday.

2) All discrete math students can tell a valid argument from an invalid one. All thoughtful people can tell a valid argument from an invalid one. Therefore all discrete math students are thoughtful.
Problem 6: Max wishes to prove the following conjecture using the method of proof by contradiction. The statement must be rewritten to fit the method. Rewrite the statement for Max. (Do not actually do the proof)
Theorem: Given integers a and b, if \((ab)^2\) is even then a is even or b is even.

In proving the theorems below, do not make use of any previously proved theorems. Use definitions only.

Problem 7: Use method by contrapositive to prove the following theorem.
Theorem: For all natural numbers n, if \(7n^2 + 5\) is even, then n is odd.

Problem 8: Use direct method to prove the following theorem:
Theorem: If \(\forall a, b, c \in \mathbb{Z}, \text{ If } ab \mid c, \text{ then } a \mid c\)

Problem 9: Use direct method to prove the following theorem.
Theorem: Every integer is a rational number.