Math 2534 Solutions Homework on Sets Fall 2013
Shows all work and justify and clarify answers

1) Let A = \{a,b,c\} , B = \{x,y\} and C = \{1,2\}
Find the following:
   a) A \times B = \{(a,x), (a,y), (b,x), (b,y), (c,x), (c,y)\}
   b) C \times B = \{(1,x), (1,y), (2,x), (2,y)\}
   c) C \times A = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}

2) Prove or give a counter example.
Theorem: For all sets A and B, A \times B = B \times A
Counter example given below.
A = \{1\} and B = \{a,b\}
A \times B = \{(1,a), (1,b)\}
B \times A = \{(a,1), (b,1)\}
Therefore A \times B \neq B \times A

3) Find the power set for :
   a) B = \{ 1,2,3,4\}
      P(B) = \{\emptyset, \{1,2,3,4\}, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\},
            \{1,2,3\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}\}
   b) A = \{\emptyset, \{\emptyset\}\}
      P(A) = \{\emptyset, \emptyset, \{\emptyset\}\}

4) Given the power set P = \{\emptyset, \{y\}\}, what was the original set?
   A = \{y\}

5) If A = \{ h, k ,a, i \} and B = \{ a, d, h, k \} find the symmetric difference A \oplus B.
   A \oplus B = (A - B) \cup (B - A) = \{i\} \cup \{d\} = \{i,d\}

6) Draw Venn Diagrams to illustrate the following:
   a) (A - B) \cup C
   b) A^C \cap B \cap C^C
   c) (A - B) \cup (B - C)

   Will do in Class
7) If the Universal set U = \{a, b, c, d, e, f, g, h\} and A = \{a, d, e, f, h\} and B = \{b, c, d, e, f, g\}
Find the following:
   a) A \oplus B = \{a, h, b, c, g\}
   b) A \cap B^C = \{a, h\}
   c) A \setminus B = \{a, h\}
   d) A^C \cup B^C = \{b, c, g, a, h\}

8) If \(A \times B = \{(a, b), (b, b), (c, b), (a, a), (b, a), (c, a)\}\), find the elements in sets A and B.
   A = \{a, b, c\} and B = \{b, a\}

9) Using elements of sets, prove the following or give a counter example.
   a) For all sets A, B and C, If A \subseteq B and C \subseteq B, then A \cup C \subseteq B
      This is actually not true since it is possible for A \cup C could equal B.
      Consider the counter example: A = \{1\}, C = \{2\}, B = \{1, 2\}.

**Correction that is valid.**
For all sets A, B and C, If A \subseteq B and C \subseteq B, then A \cup C \subseteq B
Proof:
\[
\forall x, x \in A \cup C \rightarrow x \in A \lor x \in C \quad \text{by definition for union}
\]
\[
\rightarrow x \in B \lor x \in C \quad \text{since A \subseteq B}
\]
\[
\rightarrow x \in B \lor x \in B \quad \text{since C \subseteq B}
\]
\[
\rightarrow x \in B \quad \text{by idempotent}
\]
Therefore A \cup C \subseteq B

b) For all sets A and B, \((A \cup B)^C = A^C \cap B^C\)
Proof:
\[
\forall x, x \in (A \cup B)^C \rightarrow x \notin (A \cup B) \quad \text{by definition of complement}
\]
\[
\rightarrow \sim (x \in A \cup B) \quad \text{logic negative}
\]
\[
\rightarrow \sim (x \in A \lor x \in B) \quad \text{Definitions of union}
\]
\[
\rightarrow \sim (x \in A) \land \sim (x \in B) \quad \text{DeMorgan Law in Logic}
\]
\[
\rightarrow (x \notin A) \land (x \notin B)
\]
\[
\rightarrow (x \in A^C) \land (x \in B^C) \quad \text{Definition of complement}
\]
\[
\rightarrow x \in (A^C \cap B^C) \quad \text{Definition of of intersection.}
\]
Therefore we have shown that \((A \cup B)^C \subseteq A^C \cap B^C\)
By reversing steps, it is clear that \(A^C \cap B^C \subseteq (A \cup B)^C\). Since set containment has been established, we have that \((A \cup B)^C = A^C \cap B^C\) by definition of equal sets.
c) For all set $A$ and $B$, $(A - B) = A$

This is false as shown by the following counter example:
Let $A = \{2, 3, 4\}$ and $B = \{3, 4\}$ and $A - B = \{2\}$ and not equal to $A$. 