Math 2534  Homework 8  Fall 2013

Problem 1:
Use Proof by elements to prove the following, where A, B, C and D are Sets:

1) \((A \cup B^C) \cap B = A \cap B\)
2) If \(A \subseteq C\) and \(B \subseteq D\), Then \(A \cap B \subseteq C \cap D\)

Problem 2:
Use Proof By Set Algebra to prove the following where A, B are Sets
Justify each step. Convert to union and intersection operations only.
\[ (A^C \cup (B - A))^C - A^C = A \]

Problem 3:
Using algebra of sets and given that the symmetric difference
\( A \oplus B = (A - B) \cup (B - A) \) prove that \((A - B) \cup (B - A) = (A \cup B) - (A \cap B)\)

Problem 4:
Use set containment to develop a conclusion from the following statement.
Show all your work and define your sets clearly. Justify each reasoning step.
1) No ducks waltz.
2) No officers ever decline to waltz.
3) All my poultry are ducks

Therefore ________________________________________.

Problem 5:
Given that \( n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) \)
Complete the following problem:
Five hundred T.V. viewers were surveyed to find out how many watched the following sports events: football, hockey and basketball. Two hundred and eighty five watched football, one hundred and ninety five watched hockey, one hundred and fifteen watched basketball, forty five watched both football and basketball, seventy watched football and hockey, fifty watched hockey and basketball and fifty watched none of these events.
a) How many watched football only?
b) How many watched at least two different sports on a regular basis but never watched three different sports.

Problem 6: Define a Boolean Algebra as follows:
Let \( B = \{ 1, 2, 3, 5, 6, 10, 15, 30 \} \) be the set of all positive divisors of 30 with operations defined on the set to be as follows: \( a + b = \text{LCM}(a,b) \), and the \( a \cdot b = \text{GCD}(a, b) \).

The complement is defined \( \overline{a} = \frac{30}{a} \).

a) Determine the identities for each operation.

b) Evaluate \( 5 + \overline{5} \) and \( 3 \cdot \overline{3} \) and determine if the results are correct for a Boolean algebra.

c) Determine if DeMorgan’s Law is valid for the following: \( (6 + 15) = 6 \cdot 15 \).

Problem 7:
Let \( a \) and \( b \) be elements in the Boolean Algebra \( B \) with the following defined operations. Operation 1 is \( \otimes \) with identity \( p \) and operation 2 is \( \odot \) with identity \( q \). The complement (or negative) is represented by \( \overline{a} \). Prove the following:

\[
\overline{a} \otimes [(a \otimes b) \otimes (b \odot q)] = a
\]