Math 2534 Homework 7 on PMI Spring 2015

Prove the following theorems using PMI. The write up needs to be complete by using sentences to explain and justify your presentation.

**Theorem 1:** \( \forall n \in N, \ 1 + 2 + 2^2 + \ldots + 2^{n-1} = 2^n - 1 \)

**Theorem 2:** \( \forall n \in N, \ 3 \mid (4^n - 1) \)

**Theorem 3:** \( \forall n \in N \geq ?, \ (n + 1)! > 2^{n+3} \)
(First determine the smallest value for which this theorem would be true.)

**Theorem 4:** \( \forall n \in N \geq ?, \ 2n + 3 \leq 2^n \)
(First determine the smallest value for which this theorem would be true.)

**Theorem 5:** If \( f(x) = x^2 e^x \), then \( f^{(n)}(x) = e^x[x^2 + 2nx + n(n-1)], \forall n \in N, x \in R \)