Problem 1: Let $A = \{ a, b, c, d, e\}$ and define $f : A \to A$ and $g : A \to A$ as follows.

$G(f) = \{(a,e),(b,c),(d,a),(c,b),(e,d)\}$

$G(g) = \{(a,c),(b,d),(d,c),(c,e),(e,a)\}$

a) Is $f$ a function? Is $f$ one to one? Is $f$ onto? Does $f^{-1}$ exists?
b) Is $g$ a function? Is $g$ one to one? Is $g$ onto? Does $g^{-1}$ exists?
c) Find $G(g \circ f)$. 
d) Find $G(f \circ f^{-1})$ if the inverse exists.

Problem 2:

A) Using proof by elements, prove that for all subsets $A$ and $B$ of a set $S$, where $f : S \to D$ that $f(A \cap B) \subseteq f(A) \cap f(B)$.

B) Give a counter example or the assertion that $f(A \cap B) \subseteq f(A) \cap f(B)$.

Problem 3: Is the following statement valid? Explain why or give a counter example.

Given sets $X$ and $Y$, a function $f : X \to Y$ is one to one (an injection) if and only if every element of $X$ is mapped by $f$ to exactly one element of $Y$.

Problem 4: Determine if the following statements are correct. If not, then explain why not.

a) Let $f : X \to Y$. A sufficient condition for $f(x)$ to be one to one is that $\forall y \in Y$, there is at most one $x \in X$, with $f(x) = y$.
b) $f : X \to Y$ is one to one if and only if $\forall x \in X$, there exist one and only one $y \in Y$ so that $f(x) = y$.
c) $f : X \to Y$ is onto if and only if the range and the co-domain are the same.
d) $f : X \to Y$ is onto if and only if $\forall x \in X$, $\exists y \in Y$ so that $f(x) = y$.

Problem 5: Let the function $h(x)$ map set $A$ to set $B$. Let $C$ and $D$ be disjoint subsets so that $C \cup D = A$.

Define functions $f : C \to B$ and $g : D \to B$ so that $h(x) = f(x)$ for all $x$ in $C$ and $h(x) = g(x)$ for all $x$ in $D$. Determine if the following is true or false and justify your reasoning.

(Hint: draw diagrams and look at examples)

a) If $f(x)$ and $g(x)$ are each one to one, will $h(x)$ also be one to one? Justify your conclusion.
b) If the inverse function of $h$ exist, then the inverse of $f$ must also exist.